Laudatio für Sandra Müller aus Anlass der Verleihung des ÖMG-Förderungspreises 2022

Dear Sandra, dear colleagues,

It is a great pleasure to present the scientific achievements of my close colleague, Sandra Müller, on the occasion of receiving the Förderungspreis of the Austrian Mathematical Society in 2022.

Recently, Sandra has won so many awards and prestigious grants that is getting hard to keep track of them. In 2020, Sandra received two prestigious fellowships, the FWF Elise Richter Fellowship and the L'ORÉAL Austria/ÖUK/ÖAW-Fellowship. Just recently, she was awarded the START Prize of FWF.

Her list of recent invitations to give presentations at prestigious conferences is equally enviable. In the last 2 years Sandra has received invitations to give lectures at:

1. 05 Jan 2023, Winter Meeting of the ASL with the JMM, Boston, Invited Address.
2. 29 Aug 2022, European Set Theory Conference, Turin, Plenary talk.
3. 27 Jun 2022, Logic Colloquium Reykjavik, Iceland.
4. 15 Sep 2021, 16th International Luminy Workshop in Set Theory.
5. 19 Jul 2021, Logic Colloquium Poznan, Poland.
6. 23 Jun 2021, ASL North American Annual Meeting
7. 24 Mar 2020, North American Annual Meeting of the ASL, Plenary lecture.

These are all major conferences devoted to set theory or mathematical logic.

Sandra obtained her PhD in 2016 from the University of Münster where she worked with Ralf Schindler. Her thesis was titled Pure Hybrid Mice with Finitely Many Woodin Cardinals from Levels of Determinacy, but fortunately it had nothing to do with mixing mouse genes with whatever the title may imply. No mice were injured and all set theorists were happy!

I am also interested in mice, my thesis was titled "A tale of hybrid mice", and so since we both like hybrid mice, we naturally ended up talking to each other, and the first time this happened was in 2015, at the Newton Institute, and I immediately realized that she has a bright future, and that I better work with her.

Sandra is very active and has made several deep contributions to set theory, and especially to inner model theory and the study of determinacy. She has many collaborators and works in several directions at the same time. Since there is no space to describe all of her work, I will concentrate on two outstanding contributions to descriptive inner model theory that she has made, one in her thesis and another more recently.

The subject of set theory is infinity, its goal is to discover mathematical laws or axioms of infinity that govern our use of the concept of infinite set. These set theoretic axioms tell us how to form new sets from old sets or that certain types of sets exist. However, as Gödel has demonstrated, the basic axioms of set theory, ZFC, just like any other sufficiently strong theory cannot decide all questions one may wish to ask. In particular, ZFC does not decide the Continuum Hypothesis as well as number of natural regularity questions about sets of reals. For example, it does not decide whether all projective sets of reals are Lebesgue measurable.

Here, projective sets are those that can be obtained from open sets by successively applying complementation and projection a finite number of times. The nth level of the projective hierarchy consists of those sets of reals that can be obtained from the open sets by applying at most n projections and at most n complementations. The fact that a question about such simple sets as projective sets, in fact the very natural question whether every projective set of reals is Lebesgue measurable, has no answer in ZFC is troubling as it suggests that there are basic principles of sets that are not part of ZFC, and moreover, as experience shows, there are no *obvious* such principles. In set theory, Gödel's program is the program of developing natural extensions of ZFC that decide more and more questions left undecided by ZFC.

Since Gödel outlined his program in What is Cantor's Continuum Problem, set theorists have discovered several natural hierarchy of axioms that decide plethora of natural questions left undecided by ZFC. Examples of such hierarchies are the large cardinal hierarchy and the determinacy hierarchy. The large cardinal hierarchy is a hierarchy of axioms asserting the existence of 0-1 measures on various large sets. The determinacy hierarchy is a hierarchy of axioms asserting the determinacy of various types of two player games of perfect information.

However, these hierarchies are logically incompatible. One of the prime goals of descriptive inner model theory is to find mathematical ways of amalgamating such natural hierarchies, and one way of doing this amalgamation has been via identifying natural models of the axioms of one hierarchy in natural models of axioms of another hierarchy. Sandra has made two deep contributions to this fundamental problem, the first in her thesis and the second more recently.

Sandra's thesis was a tour de force. She set out to complete one of the most difficult unpublished proofs in set theory. In the 1990s, Hugh Woodin (Harvard), John Steel (UC Berkeley) and later Itay Neeman (UCLA) developed tools for establishing level-by-level equivalencies between determinacy of two-player games whose payoff set is a projective set of reals and the existence of canonical models of set theory that satisfy large cardinal hypothesis. While this work is among the utmost achievements of the subject, Woodin never published what he thought was a complete proof. Sandra took it upon herself to complete the proof; in fact, as the story goes, the idea was to simply learn inner model theory by "writing up" what was supposed to be a well-known proof. It turned out to be nothing of that sort. The project had many ups and downs, and required many new ideas. Finally, in collaboration with Woodin and her PhD adviser Ralf Schindler they finished the project and produced a 100 page-long masterpiece. The main result of the paper is a sharp Transfer Theorem, that transfers a determinacy hypothesis into the language of canonical models of large cardinals. Putting a theorem of Neeman and the aforementioned theorem of Sandra and collaborators togather, we obtain a results that links the determinacy of all sets of reals in the nth projective hierarchy with the existence of an inner model with n Woodin cardinals.

Sandra's second major contribution is of a similar nature but first of all, it is a solo effort and second of all, it establishes a link between much more complicated determinacy axiom and an axiom in the large cardinal hierarchy. Her second result has the prospect of generalizing all the way and establishing level by level bridges between determinacy hierarchy and the large cardinal hierarchy. In another tour de force, she proves that the determinacy axiom asserting first that every set of reals is determined and second that every set of reals is universally Baire (i.e., its preimages have the property of Baire in all topological spaces) is equiconsistent with a large cardinal axiom asserting the existence of a cardinal which is a limit of Woodin cardinals and strong cardinals. To prove the theorem, Sandra generalizes techniques which have been used by John Steel, myself and others to translate some strong determinacy hypothesis over to the realm of large cardinals. Sandra's main ingenious contribution is a way of dovetailing this technique to make it work at limit steps of such translations.

I highly value this theorem, I myself have tried to do it, and have conjectured that it can be done. Thank you Sandra for solving this problem.

Nowadays, Sandra is continuing her adventure with hybrid mice, translating them into large cardinals, then translating them back, iterating them, computing their derived models, but never injuring any of them. I am very grateful for having such a wonderful colleague to work with and discuss mathematics. Undoubtedly, she has one of the brightest futures in set theory.

Congratulations on this major success! You fully deserve it.

(Grigor Sargsyan)