Second Austrian Stochastics Day

satellite conference of the ÖMG-DMV congress

Abstracts

September 27, 2013

Innsbruck, Austria

Welcome to Innsbruck

The organizers

Evelyn Buckwar (University of Linz)

Christel Geiss (University of Innsbruck)

Erika Hausenblas (Montanuniversity Leoben)

Printed at the University of Innsbruck, 2013

Talks

Markus Ableidinger

Johannes Kepler University Linz

Numerical Methods for the Stochastic Landau-Lifshitz-Gilbert Equation

The Stochastic Landau-Lifshitz-Gilbert (SLLG) equation describes the magnetization of a ferromagnetic material occupying a bounded domain $D \subset \mathbb{R}^n, n = 2, 3$, where the direction is subject to thermal fluctuations.
Given a complete probability space $(\Omega, \mathcal{F}_t, \mathcal{F}, P)$ with filtration $\mathcal{F}_t$ and a Hilbert space $\mathcal{K}$, the SLLG takes the form

$$\begin{aligned} dm(t,x) = m(t,x) \times [\Delta \quad m(t,x) - \alpha m(t,x) \times \Delta m(t,x)]dt \\ +\nu m(t,x) \times \circ dW(t,x) \end{aligned} \tag{1}$$

with

$$\begin{aligned} \partial_n m(t,x) &= 0 \qquad && \forall (t,x) \in \mathbb{R}^+ \times \partial D \\ m(0,x) &= m_0(x) && \forall x \in D. \end{aligned}$$

W is a $\mathcal{K}-$valued Wiener process, $\alpha, \nu \in \mathbb{R}$ and $\circ$ denotes a Stratonovich-type integral. An important requirement for a successful numerical treatment of (1) is that the numerical methods respect the qualitative behaviour of the solution of (1), for example the conservation of the modulus

$$|m| = 1.$$

In this talk we will discuss numerical methods for solving the corresponding system of SODEs after spatial discretisation. Our main focus will be on time integration schemes, which preserve the geometric structure of the SLLG, for example implicit midpoint methods and invariant preserving stochastic Runge-Kutta methods.

Florian Baumgartner

University of Innsbruck

Invariant Lévy random variables

We investigate subspaces of the Hilbert space of square integrable random variables measurable with respect to the σ-algebra generated by a Lévy process. These subspaces are characterized by invariances under groups of time permutations of the Lévy process. Our approach uses the Wiener-Itô chaos expansion for Lévy random variables which provides a completely deterministic treatment of the problem. We obtain ergodic-type theorems depending on conditions on the structure of the permutation group. By this procedure all of these invariant subspaces of random variables can be endowed with a suitable *reduced* chaos decomposition. This talk is based on joint work with S. Geiss.

Goncalo dos Reis

Technische Universität Berlin

FBSDEs with non-Lipschitz drivers and probabilistic schemes for reaction-diffusion PDEs

We undertake the error analysis of the time discretization of systems of Forward-Backward Stochastic Differential Equations (FBSDEs) with drivers having polynomial growth in the state variable and that are also monotone.

Contrary to the canonical Lipschitz driver case, we show with a counter-example that the natural explicit Euler scheme diverges! This divergence is due to the lack of a certain stability property which is essential to obtain convergence. However, a thorough analysis of the family of θ-schemes reveals that this required stability property can be recovered if the scheme is sufficiently implicit. As a by-product of our analysis we shed some light on higher order approximation schemes for FBSDEs under non-Lipschitz condition. We then return to fully explicit schemes and show that an appropriately tamed version of the explicit Euler scheme enjoys the required stability property and as a consequence it converges.

In order to establish convergence of the several discretizations we study, we extend the canonical path- and first order variational regularity results to FBSDEs with polynomial monotone drivers. These results are of independent interest for the theory of FBSDEs.

Erika Hausenblas

Montanuniversität Leoben

Controllability and qualitative properties of the solutions to SPDEs driven by boundary noise

Let u be the solution to the following stochastic evolution equation

$$\left\{ \begin{array}{rcl} du(t,x) & = & Au(t,x)\,dt + B\,\sigma(u(t,x))\,dL(t), \quad t>0; \\ u(0,x) & = & x \end{array} \right. \tag{1}$$

taking values in an Hilbert space H, where L is a $\mathbb{R}$ valued Lévy process, $A: H \to H$ an infinitesimal generator of a strongly continuous semigroup, $\sigma : H \to \mathbb{R}$ bounded from below and Lipschitz continuous, and $B : \mathbb{R} \to H$ a possibly unbounded operator. A typical example of such an equation is a stochastic partial differential equation with boundary Lévy noise. Let $\mathcal{P} = (\mathcal{P}_t)_{t\ge 0}$ be the corresponding Markovian semigroup.

We show that, if the system

$$\left\{ \begin{array}{rcl} du(t) & = & Au(t)\,dt + B\,v(t)\,dt, \quad t>0; \\ u(0) & = & x \end{array} \right. \tag{2}$$

is approximate controllable in time $T > 0$, with control v, then under some additional conditions on B and A, for any $x \in H$ the probability measure $\mathcal{P}_T^\star \delta_x$ is positive on open sets of H. Secondly, we investigate under which conditions on the Lévy process L and on the operators A and B the solution to Equation (1) is asymptotically strong Feller, respectively, has a unique invariant measure. We apply these results to the damped wave equation driven by Lévy boundary noise.

Harald Hinterleitner

Johannes Kepler University Linz

Modelling with the stochastic neural field equation

Neural field equations provide a useful framework for modelling macroscopic neural dynamics on the cortex involving a spatially distributed population of neurons.

The deterministic neural field equation is a nonlinear integro-differential equation of the form

$$\frac{\partial u(x,t)}{\partial t} + \alpha u(x,t) = \int_{\mathcal{D}} w(x,y,t) f(u(y, t - \bar{d}(x,y)))dy + I_{ext}(x,t), \tag{3}$$

where $x \in \mathcal{D} \subset \mathbb{R}^d$ with $d = 1, 2, 3$ and $t > 0$. The appropriate initial condition reads as

$$u(x,0) = u_0(x).$$

This type of equation may serve (and is applied in the neuroscience literature) as underlying models to interpret electroencephalography (EEG) and magnetoencephalography (MEG) data in certain cases.

In this talk I will present some background on the modelling assumptions and possible extensions to stochastic versions of the dNFE (3). The aim of my work will be to establish a framework of nonlinear filtering in order to estimate the state process from EEG data. First steps in this direction are proposed here.

Peter Imkeller

Humboldt Universität Berlin

A Fourier analytic approach to rough paths

In 1961, Ciesielski established a remarkable isomorphism of spaces of Hölder continuous functions and Banach spaces of real valued sequences. This isomorphism leads to wavelet decompositions of Gaussian processes giving access for instance to a precise study of their large deviations, as shown by Baldi and Roynette. We will use Schauder representations for a pathwise approach of integration, using Ciesielski's isomorphism. It can be formulated in terms of dyadic martingales and Rademacher functions. In a more general and analytical setting, this pathwise approach of rough path analysis can be understood in terms of Paley-Littlewood decompositions of distributions, and Bony paraproducts in Besov spaces. This talk is based on work in progress with M. Gubinelli (U Paris-Dauphine) and N. Perkowski (HU Berlin).

CHRISTIAN IRRGEHER

Johannes Kepler University Linz

Adapting quasi-Monte Carlo methods to simulation problems

The computation of expected values of functions depending on Brownian motions occurs in many applications. For example, in finance the valuation of financial derivatives can quite often be reduced to such a problem. A possible way to tackle this problem is to use simulation techniques like quasi-Monte Carlo methods (QMC).

To place special emphasis on the construction method used for the discrete Brownian path makes sense for quasi-Monte Carlo simulation, because it can have a big influence on the efficiency of QMC. Every Brownian path construction method, e.g., Brownian bridge construction or principal component analysis construction, can be represented by an orthogonal transform. Thus, one can use orthogonal transforms to adapt QCM to the underlying problem.

In this talk we will discuss the effect of orthogonal transforms on QMC. Therefore, we set up a Korobov-type space of functions on $\mathbb{R}^d$ based on Hermite polynomials to perform an error analysis and give tractability results.

ASGAR JAMNESHAN

Humboldt Universität Berlin

Conditional Sets

We give an introduction into the elements of conditional set theory and topology, and motivate the theory by applications into mathematical finance. This talk is based on joint works with Samuel Drapeau, Martin Karliczek and Michael Kupper.

Conall Kelly

University of the West Indies

Response to low-intensity stochastic perturbation in non-normal dynamical systems

The equilibrium of a linear system with non-normal coefficient matrix may display a large transient response to initial-value perturbations even when the equilibrium is asymptotically stable. Such transients render the stability of the equilibrium vulnerable to low-intensity, state-dependent stochastic perturbation; see Higham & Mao [1] for a theoretical example.
This phenomenon arises in, for example, certain models in ecology, fluid dynamics, and system control; and it is therefore relevant to the stability analysis of numerical methods for stochastic differential equations [2]. We will look at the effect of varying perturbation geometries on the mean-square dynamics of stochastic systems arising in such applications, and discuss potential directions for future research.
The work presented is jointly conducted with Prof. Evelyn Buckwar (Johannes Kepley University, Linz, Austria).

References

[1] D. J. Higham and X. Mao, *Nonnormality and stochastic differential equations,* BIT, 46 (2006), pp. 525-532.

[2] E. Buckwar and C. Kelly, *Non-normal drift structures and linear stability analysis of numerical methods for systems of stochastic differential equations,* Comput. Math. Appl., 64 (2012), pp. 2282-2293.

Gunther Leobacher

Johannes Kepler University Linz

(Non-)existence of strong solutions of SDEs with discontinuous drift coefficient

The classical existence result by Itô for the existence of strong solutions of stochastic differential equations (SDEs) with Lipschitz coefficients can be extended to the case where the drift is only measurable and bounded, a result by Zvonkin (1974) and Veretennikov (1981). However, those results rely on the uniform ellipticity of the diffusion coefficient.

We study the case of degenerate ellipticity. SDEs with this property arise from control/filtering problems, for example the problem of optimal dividend payment of a corporation in a Markov switching economy.

Besides showing how one can prove existence results for the degenerate elliptic case, we present examples illustrating the difficulties in obtaining more general results than those given.

The talk is based on joint work with Stefan Thonhauser and Michaela Szölgyenyi.

References

[1] A. JU. Veretennikov, *On strong solutions and explicit formulas for solutions of stochastic integral equations.* Math. USSR Sbornik, 39(3), 1981.

[2] A. K. Zvonkin, *A transformation of the phase space of a diffusion process that removes the drift.* Math. USSR Sbornik, 22(1), 1974.

Tijana Levajković

University of Belgrade

Stochastic Dirichlet problem driven by the Ornstein-Uhlenbeck operator

This talk is devoted to the stochastic version of the Fredholm alternative in the framework of chaos expansion methods on white noise probability space. We apply the results to solve the Dirichlet problem generated by an elliptic second order differential operator with stochastic coefficients, stochastic input data and boundary conditions, and with the Ornstein-Uhlenbeck operator as a perturbation term.

References

[1] T. Levajković, S. Pilipović, D. Seleši, *The stochastic Dirichlet problem driven by the Ornstein-Uhlenbeck operator: Approach by the Fredholm alternative for chaos expansions*, Stochastic Analysis and Applications, Vol. 29, pp 317–331, 2011.

[2] S. Pilipović, D. Seleši, *On the stochastic Dirichlet problem – Part I: The stochastic weak maximum principle*, Potential Analysis, 32(4), pp 363–387, 2010.

Alexander Steinicke

University of Innsbruck

Crucial steps in Malliavin differentiation of Lévy driven BSDEs

We consider a Lévy process $X = (X_t)_{t\in[0,T]}$ with Lévy measure ν and Lévy-Itô decomposition

$$X_t = \gamma t + \sigma W_t + \int_{(0,t]\times\{1<|x|\}} xN(ds,dx) + \int_{(0,t]\times\{0<|x|\le 1\}} x\tilde{N}(ds,dx),$$

where $\gamma \in \mathbb{R}, \sigma \geq 0$ are constants, (W_t) denotes the Brownian part, N and $\tilde{N}$ are the Poisson random measure and the compensated Poisson random measure, respectively. Let $\mu(dx) := \sigma^2\delta_0(dx) + x^2\nu(dx)$ and let $\kappa \in L_2(\mathbb{R},\mu)$.

We investigate the BSDE (backward stochastic differential equation)

$$Y_t = \xi + \int_t^T f\left(s, (X_r)_{0\le r\le s}, Y_s, \int_{\mathbb{R}} Z_{s,x}\kappa(x)\mu(dx)\right) ds - \int_{(t,T]\times\mathbb{R}} Z_{s,x}M(ds,dx),$$

with $0 \le t \le T$, where the random measure M is given by

$$M(ds,dx) := \sigma dW_t\delta_0(dx) + x\tilde{N}(dt,dx).$$

We discuss certain steps of the Malliavin differentiation of the BSDE under assumptions as:

- The terminal condition ξ is Malliavin differentiable
- The generator $f\left(s, (X_r)_{0\le r\le s}, y, z\right)$ is continuously differentiable in (y,z)
- Malliavin differentiability of $f(s, (X_r)_{0\le r\le s}, y, z)$

We further investigate how additional regularity assumptions on f w.r.t. the variables y, z behave under Malliavin differentiation and prove the equation's differentiability in case of a Lipschitz assumption.

Malliavin differentiation of BSDEs is important to access the Z process explicitly from the Y process. Moreover it is an essential tool for investigating smoothness properties of BSDEs. This is joint work with C. Geiss.

Christoph Temmel

Vrije Universiteit Amsterdam

Lumped Markov chains and entropy rate preservation

(Joint with Bernhard C. Geiger) A lumping of a Markov chain is a coordinate-wise projection of the chain, also known as a hidden Markov chain. We characterise the entropy rate loss of a lumping of a stationary Markov chain on a finite state space in two ways. First, by the asymptotic ratio of the number of trajectories with positive weight between the original and the lumped chain. Second, by the reconstructability of original trajectories from their images under the lumping. Every non-trivial lumping of a Markov chain with positive transition matrix incurs an entropy rate loss. We give sufficient conditions on the non-positive transition matrix and the lumping to preserve the entropy rate. We also characterise the strong k-lumpability of the lumping by tight entropic bounds in the stationary setting.

Anton Wakolbinger

Goethe-Universität Frankfurt am Main

From 0 to 100 in a spatial sweep: a graphical stochastic analysis of a noisy logistic system

The system in the title describes the evolution of the vector of relative frequencies of a beneficial allele in d colonies, starting in (0,...,0) and ending in (1,...,1). Its diffusion part consists of Wright-Fisher noises in all the components that model the random reproduction, its drift part consists of a linear interaction term coming from the gene flow between the colonies, together with a logistic growth term due to the selective advantage of the allele, and a term which makes the entrance from (0,...,0) possible. It turns out that there are d extremal ones among the solutions of the system, each of them corresponding to one colony in which the beneficial mutant originally appears. Of special interest is the time it takes for the beneficial allele in the limit of a large selection coefficient to 'sweep through the population'. We explain how the asymptotic distribution of this fixation time can be analysed in terms of the so called ancestral selection graph. This is joint work with Andreas Greven, Peter Pfaffelhuber and Cornelia Pokalyuk.

JUHA YLINEN

University of Jyväskylä

Anisotropic Besov spaces in the Wiener space

In this talk we work in the Wiener space with a Brownian Motion $(W_t)_{t\in[0,T]}$. First we introduce a decoupling method as follows: first, take any $\varphi : [0,T] \to [0,1]$, and define a new Brownian Motion by

$$W_t^\varphi := \int_0^t \sqrt{1-\varphi^2(r)}dW_r + \int_0^t \varphi(r)dB_r,$$

where B is a Brownian Motion independent of W. Then, taking any random variable ξ, we may define ξ^φ by switching the randomness of ξ to come from W^φ instead of W. For $p \geq 1$ and $\xi \in L_p(\Omega)$ we will measure the quantity $\|\xi - \xi^\varphi\|_p$ in different ways, and this gives rise to different Anisotropic Besov spaces. These spaces include for example the classical Besov spaces obtained by the real interpolation method, but also other examples and applications will be considered. This is a joint work with S. Geiss.

REFERENCES

[1] D. Nualart: The Malliavin Calculus and Related Topics. 2nd edition, Springer, 1995.

Participants

1 **Markus Ableidinger** (Johannes Kepler University Linz, Austria)
2 **Florian Baumgartner** (University of Innsbruck, Austria)
3 **Evelyn Buckwar** (Johannes Kepler University Linz, Austria)
4 **Elisabetta Candellero** University of Birmingham, United Kingdom
5 **Goncalo dos Reis** (Technische Universität Berlin, Germany)
6 **Tina Engler** (Martin-Luther University Halle-Wittenberg, Germany)
7 **Christel Geiss** (University of Innsbruck, Austria)
8 **Stefan Geiss** (University of Innsbruck, Austria)
9 **Martin Grothaus** (Technische Universität Kaiserslautern, Germany)
10 **Erika Hausenblas** (Montanuniversitaet Leoben, Austria)
11 **Harald Hinterleitner** (Johannes Kepler University Linz, Austria)
12 **Wilfried Huss** (Graz University of Technology, Austria)
13 **Peter Imkeller** (Humboldt University Berlin, Germany)
14 **Christian Irrgeher** (Johannes Kepler University Linz, Austria)
15 **Asgar Jamneshan** (Humboldt University Berlin, Germany)
16 **Conall Kelly** (University of the West Indies, Jamaica)
17 **Nikolai Kolev** (University of Sao Paulo, Brazil)
18 **Gunther Leobacher** (Johannes Kepler University Linz, Austria)
19 **Tijana Levajkovic** (University of Belgrade, Serbia)
20 **Ahmed Sani** (Université Ibn Zohr, Morocco)
21 **Ecaterina Sava-Huss** (Graz University of Technology, Austria)
22 **Alexander Steinicke** (University of Innsbruck, Austria)
23 **Michaela Szölgyenyi** (Johannes Kepler University Linz, Austria)
23 **Christoph Temmel** (Vrije Universiteit Amsterdam, Netherlands)
25 **Elmar Teufl** (Eberhard Karls Universität Tübingen, Germany)
25 **Robert Tichy** (Graz University of Technology, Austria)
27 **Reinhard Viertl** (Vienna University of Technology, Austria)
28 **Dominik Vu** (University of Memphis, USA)
29 **Anton Wakolbinger** (Goethe University Frankfurt am Main, Germany)
30 **Martin Wendler** (Ruhr-Universität Bochum, Germany)
31 **Wolfgang Woess** (Graz University of Technology, Austria)
32 **Juha Ylinen** (University of Jyväskylä, Finland)
33 **Sabine Bormann** (Maplesoft GmbH, Germany)
34 **Thomas Richard** (Maplesoft GmbH, Germany)