Internationale Mathematische Nachrichten
International Mathematical News
Nouvelles Mathématiques Internationales

Die IMN wurden 1947 von R. Inzinger als „Nachrichten der Mathematischen Gesellschaft in Wien“ gegründet. 1952 wurde die Zeitschrift in „Internationale Mathematische Nachrichten“ umbenannt und war bis 1971 offizielles Publikationsorgan der „Internationalen Mathematischen Union“.

Von 1953 bis 1977 betreute W. Wunderlich, der bereits seit der Gründung als Redakteur mitwirkte, als Herausgeber die IMN. Die weiteren Herausgeber waren H. Vogler (1978–79), U. Dieter (1980–81, 1984–85), L. Reich (1982–83), P. Flor (1986–99), M. Drmota (2000–2007) und J. Wallner (2008–2017).

Herausgeber:

Österreichische Mathematische Gesellschaft, Wiedner Hauptstraße 8–10/104, A-1040 Wien. email *imn@oemg.ac.at*, *http://www.oemg.ac.at/*

Bezug:

Die IMN erscheinen dreimal jährlich und werden von den Mitgliedern der Österreichischen Mathematischen Gesellschaft bezogen.

Jahresbeitrag: € 35,–

Bankverbindung:
IBAN AT83-1200-0229-1038-9200 bei der Bank Austria-Creditanstalt (BIC-Code BKAUATWW).

Eigentümer, Herausgeber und Verleger: Österr. Math. Gesellschaft. Satz: Österr. Math. Gesellschaft. Druck: Weinitzendruck, 8044 Weinitzen.

ISSN 0020-7926

Österreichische Mathematische Gesellschaft

Gegründet 1903
http://www.oemg.ac.at/
email: *oemg@oemg.ac.at*

Sekretariat:

Alpen-Adria-Universität Klagenfurt,
Institut für Mathematik
Universitätsstraße 65-67
A-9020 Klagenfurt
email: *oemg@oemg.ac.at*

Vorstand des Vereinsjahres 2021:

B. Kaltenbacher (Univ. Klagenfurt): Vorsitzende
J. Wallner (TU Graz): Stellvertretender Vorsitzender
C. Fuchs (Univ. Salzburg): Herausgeber der IMN
M. Ludwig (TU Wien): Schriftführerin
M. Haltmeier (Univ. Innsbruck): Stellvertretender Schriftführer
B. Lamel (Univ. Wien): Kassier
P. Grohs (Univ. Wien): Stellvertretender Kassier
E. Resmerita (Univ. Klagenfurt): Beauftragte für Frauenförderung
C. Heuberger (Univ. Klagenfurt): Beauftragter f. Öffentlichkeitsarbeit

Beirat:

A. Binder (Linz)
M. Drmota (TU Wien)
H. Edelsbrunner (ISTA)
H. Engl (Univ. Wien)
H. Heugl (Wien)
W. Imrich (MU Leoben)
M. Kim (MathWorks)
M. Koth (Univ. Wien)
M. Kraker (Graz)
C. Krattenthaler (Univ. Wien)
W. Müller (Univ. Klagenfurt)
H. Niederreiter (ÖAW)
W. G. Nowak (Univ. Bodenkultur)
M. Oberguggenberger (Univ. Innsbruck)
W. Schachermayer (Univ. Wien)
K. Sigmund (Univ. Wien)
H. Sorger (Wien)
R. Tichy (TU Graz)
K. Unterkofler (FH Dornbirn)
H. Zeiler (Wien)

Vorsitzende von Sektionen und Kommissionen:

W. Woess (Graz)
H.-P. Schröcker (Innsbruck)
C. Heuberger (Klagenfurt)
F. Pillichshammer (Linz)
S. Blatt (Salzburg)
I. Fischer (Wien)
H. Humenberger (Didaktikkommission)
W. Müller (Verantwortlicher für Entwicklungszusammenarbeit)

Die Landesvorsitzenden und der Vorsitzende der Didaktikkommission gehören statutengemäß dem Beirat an.

Mitgliedsbeitrag:

Jahresbeitrag: € 35,–

Bankverbindung: IBAN AT83-1200-0229-1038-9200

Internationale Mathematische Nachrichten

International Mathematical News

Nouvelles Mathématiques Internationales

Nr. 247 (75. Jahrgang) **August 2021**

Inhalt

Gregor Gantner: On numerical quadrature of singular boundary integrals . 1

Norwegian Academy of Science and Letters: Abel Prize 2021 11

Franz Lemmermeyer: Ein Brief von Eisensteins Eltern an Gauß 19

Karl Kunisch: Franz Kappel 1940–2020 29

Joachim Escher: Nonlinear Water Waves: A Challenge for Mathematics . 31

Rudolf Taschner: Nicht was real ist, sondern was bleibt, sollte gefragt werden 35

Buchbesprechungen 39

Women in Mathematics 41

Nachrichten der Österreichischen Mathematischen Gesellschaft 45

Neue Mitglieder 47

Die Titelseite zeigt die epidemiologische Kurve von Covid-19 in Österreich im ersten Jahr der Pandemie. Die Daten stammen vom Dashboard der AGES. In diesem Jahr wurde "flatten the curve" zu einem viel zitierten Schlagwort, die effektive Reproduktionszahl, die 7-Tage-Inzidenz und andere Kennwerte sind einer informierten Öffentlichkeit nun bestens vertraut, exponentielles Wachstum ist für alle greifbarer geworden. In der Krise haben Mathematikerinnen und Mathematiker wichtige Sichtweisen beigesteuert und gemeinsam mit anderen Expertinnen und Experten somit einen wesentlichen Input für die Entscheidungen der Politik geliefert. Die Krise hat – so wie in allen Bereichen des Lebens – auch in der mathematischen scientific community für etliche Änderungen gesorgt. Videokonferenzen haben unseren Austausch geprägt. Offenbar geht es doch auch ohne Tafel, wenngleich die Diskussion und die Vermittlung von Ideen dabei gelitten hat! Es besteht die berechtigte Hoffnung, dass die gesundheitliche Gefahr mit der Impfung deutlich reduziert wird, sodass die sehr herausfordernden und schwierigen Pandemie-Monate dann letztlich hinter uns liegen werden.

Internat. Math. Nachrichten
Nr. 247 (2021), 1–10

On numerical quadrature of singular boundary integrals

Gregor Gantner
University of Amsterdam

About the author: Gregor Gantner studied mathematics at TU Wien (Master 2014, PhD *sub auspiciis Praesidentis rei publicae* supervised by Dirk Praetorius in 2017). His PhD thesis was awarded with the *Studienpreis der ÖMG* as well as the *Dr.-Klaus-Körper-Preis der GAMM*. He is currently postdoctoral researcher at the Korteweg-de Vries Institute for Mathematics at the University of Amsterdam funded by the Austrian Science Fund (FWF) through the *Erwin-Schrödinger-Auslandsstipendium* J4379-N.

1 Introduction

For a two-dimensional bounded domain $\Omega \subset \mathbb{R}^2$ whose boundary $\Gamma := \partial\Omega$ can be parametrized via some continuous piecewise smooth path $\gamma : [0,1] \to \Gamma$ with $|\gamma'| > 0$ and $\gamma(0) = \gamma(1)$, we consider the Laplace model problem with given Dirichlet data g, i.e.,

$$\begin{aligned} \Delta u &= 0 \quad \text{in } \Omega, \\ u &= g \quad \text{on } \Gamma. \end{aligned} \tag{1}$$

Here Δ denotes the Laplace operator $\partial_{x_1}^2 + \partial_{x_2}^2$. For

$$g \in H^{1/2}(\Gamma) := \big\{ h \in L^2(\Gamma) : \int_\Gamma \int_\Gamma \frac{|h(x) - h(y)|}{|x-y|} \,\mathrm{d}y\,\mathrm{d}x < \infty \big\},$$

it is well-known that (1) exhibits a unique solution, which can be given explicitly by solving a boundary integral equation:

ISSN 0020-7926

Abbreviate $G(x,y) := -\frac{1}{2\pi}\log|x-y|$. For $\phi \in L^\infty(\Gamma)$, the *single-layer potential*

$$(\widetilde{\mathfrak{V}}\phi)(\widetilde{x}) := \int_\Gamma G(\widetilde{x},y)\phi(y)\,\mathrm{d}y \quad \text{for } \widetilde{x} \in \Omega$$

and the *single-layer operator*

$$(\mathfrak{V}\phi)(x) := \int_\Gamma G(x,y)\phi(y)\,\mathrm{d}y \quad \text{for } x \in \Gamma$$

are both well-defined. Identifying ϕ with the dual functional $v \mapsto \int_\Gamma \phi(y)v(y)\,\mathrm{d}y \in H^{-1/2}(\Gamma) := H^{1/2}(\Gamma)'$, they can be continuously extended to operators $\widetilde{\mathfrak{V}} : H^{-1/2}(\Gamma) \to \{v \in C^1(\Omega) \cap L^2(\Omega) : \nabla v \in L^2(\Omega)\}$ and $\mathfrak{V} : H^{-1/2}(\Gamma) \to H^{1/2}(\Gamma)$. If $\phi \in H^{-1/2}(\Gamma)$ satisfies the boundary integral equation

$$\mathfrak{V}\phi = g, \tag{2}$$

then the solution u of (1) is explicitly given by $u = \widetilde{\mathfrak{V}}\phi$. If $\mathrm{diam}(\Omega) < 1$ (which can always be attained after rescaling), $(\phi,\psi) \mapsto \langle \mathfrak{V}\phi\,,\,\psi\rangle$ defines an equivalent scalar product on the Hilbert space $H^{-1/2}(\Gamma)$, where $\langle\cdot\,,\,\cdot\rangle$ denotes the duality pairing between $H^{1/2}(\Gamma)$ and $H^{-1/2}(\Gamma)$. Hence, the Riesz theorem guarantees unique solvability of (2). For more details and proofs, the interested reader is referred to the monographs [10, 14, 13].

Given some finite-dimensional space $\mathcal{X} \subset L^\infty(\Gamma)$, e.g., the space of piecewise constants with respect to some boundary mesh, the goal of the *boundary element method* (BEM) is to approximate ϕ by some function $\Phi \in \mathcal{X}$ and subsequently $u = \widetilde{\mathfrak{V}}\phi$ by $U := \widetilde{\mathfrak{V}}\Phi$. The so-called *Galerkin* BEM computes Φ as orthogonal projection with respect to the scalar product $(\phi,\psi) \mapsto \langle \mathfrak{V}\phi\,,\,\psi\rangle$, i.e., $\langle \mathfrak{V}\Phi\,,\,\Psi\rangle = \langle g\,,\,\Psi\rangle$ for all $\Psi \in \mathcal{X}$, or more explicitly,

$$\int_\Gamma\int_\Gamma G(x,y)\Phi(y)\Psi(x)\,\mathrm{d}y\,\mathrm{d}x = \int_\Gamma g(y)\Psi(y)\,\mathrm{d}y \quad \text{for all } \Psi \in \mathcal{X}. \tag{3}$$

Alternatively, *collocation* BEM chooses $N := \#\mathcal{X}$ different points $(x_i)_{i=1}^N$ on the boundary Γ and tries to solve

$$(\mathfrak{V}\Phi)(x_i) = \int_\Gamma G(x_i,y)\Phi(y)\,\mathrm{d}y = g(x_i) \quad \text{for all } i \in \{1,\ldots,N\}. \tag{4}$$

While in contrast to Galerkin BEM, existence (and thus uniqueness) of such a $\Phi \in \mathcal{X}$ is actually not guaranteed, collocation BEM is still very popular among engineers due to its simplicity.

In the present manuscript, we explain how to numerically compute the involved singular integrals of Galerkin as well as collocation BEM provided that $\mathcal{X}$ consists of piecewise smooth functions on the boundary. The presented techniques have been used in the recent own works [6, 5, 8, 7, 9] for so-called *isogeometric* functions [2, 12]. Transformed onto the parameter domain of γ, the latter are piecewise rational functions with respect to some partition $\widehat{\mathcal{Q}}$ of $[0,1]$ into finitely many closed intervals (where the involved denominators are non-zero on these intervals). The corresponding boundary mesh is denoted by $\mathcal{Q}$. An alternative method to compute the integrals for isogeometric functions is found in [1].

The mentioned own works all investigate the convergence behavior of *adaptive* BEM, which iteratively refines the boundary mesh enriching the corresponding approximation space $\mathcal{X}$ and computes the Galerkin or collocation solution therein. To steer this adaptive algorithm, a suitable *a posteriori* computable error estimator is required that locally indicates where the error is large. The *Faermann* estimator [3, 4] is one of such estimators,

$$\eta^2 := \sum_{Q\in\mathcal{Q}} \eta(Q)^2 \text{ with } \eta(Q)^2 := \sum_{\substack{Q'\in\mathcal{Q}\setminus\{Q\}\\ Q\cap Q'\neq\emptyset}} \int_{Q\cup Q'}\int_{Q\cup Q'} \frac{|r(x)-r(y)|^2}{|x-y|^2}\,\mathrm{d}y\,\mathrm{d}x, \quad (5)$$

where $r := g - \mathfrak{V}\Phi$ abbreviates the residual. For isogeometric spaces $\mathcal{X}$, it satisfies *efficiency* as well as *reliability*

$$C_{\mathrm{eff}}^{-1}\,\eta^2 \leq \|\phi-\Phi\|^2_{H^{-1/2}(\Gamma)} \leq C_{\mathrm{rel}}\,\eta^2,$$

where $C_{\mathrm{eff}}, C_{\mathrm{rel}} \geq 1$ are constants that remain unchanged under (admissible) mesh-refinement; see [6]. Using the techniques of Galerkin and collocation BEM, we also show how to compute η.

2 Galerkin BEM

Given some basis $(\Phi_i)_{i=1}^N$ of $\mathcal{X}$, the Galerkin BEM approximation $\Phi = \sum_{i=1}^N c_i\Phi_i$ of (3) can be computed by solving the linear system

$$\big(\langle\mathfrak{V}\Phi_j\,,\,\Phi_i\rangle\big)_{i,j=1}^N \cdot \big(c_i\big)_{i=1}^N = \big(\langle g\,,\,\Phi_i\rangle\big)_{i=1}^N.$$

We show how to compute $\langle\mathfrak{V}\phi\,,\,\psi\rangle$ as well as $\langle g\,,\,\psi\rangle$ for (at least) $\mathcal{Q}$-piecewise continuously differentiable path γ and (at least) $\mathcal{Q}$-piecewise continuous functions ϕ, ψ, g on the boundary.

The numerical computation of the right-hand side is simple. We start with

$$\langle g\,,\,\psi\rangle = \int_\Gamma g(x)\psi(x)\,\mathrm{d}x = \sum_{Q\in\mathcal{Q}}\int_Q g(x)\psi(x)\,\mathrm{d}x.$$

Let $Q \in \mathcal{Q}$ with $Q = \gamma(\widehat{Q})$, $\widehat{Q} = [a,b]$ for some $a,b \in [0,1]$. Then, elementary integral transformations show that

$$\begin{aligned}\int_Q g(x)\psi(x)\,\mathrm{d}x &= \int_{\widehat{Q}} g\big(\gamma(s)\big)\psi\big(\gamma(s)\big)|\gamma'(s)|\,\mathrm{d}s \\ &= \int_0^1 g\big(\gamma_Q(\sigma)\big)\psi\big(\gamma_Q(\sigma)\big)|\gamma_Q'(\sigma)|\,\mathrm{d}\sigma,\end{aligned}$$

where $\gamma_Q(\sigma) := \gamma\big(a+\sigma(b-a)\big)$. The last term has the form $\int_0^1 f(\sigma)\,\mathrm{d}\sigma$ for some (at least) continuous function f on $[0,1]$. To numerically approximate such integrals, one chooses different points $(\sigma_k)_{k=1}^n$ on $[0,1]$ and replaces f by its interpolation polynomial of degree $n-1$. Choosing $(\sigma_k)_{k=1}^n$ as the so-called *Gaussian quadrature points* [11, Chapter 6] and letting $n \to \infty$ results in convergence of the approximation to the exact integral, where the convergence is even exponential with respect to the number of points if the integrand is smooth.

For the left-hand side $\langle \mathfrak{V}\phi\,,\,\psi\rangle$, we start with

$$\langle \mathfrak{V}\phi\,,\,\psi\rangle = \int_\Gamma\int_\Gamma \phi(y)\psi(x)G(x,y)\,\mathrm{d}y\,\mathrm{d}x = \sum_{Q\in\mathcal{Q}}\sum_{Q'\in\mathcal{Q}}\int_Q\int_{Q'}\phi(y)\psi(x)G(x,y)\,\mathrm{d}y\,\mathrm{d}x.$$

Let $Q, Q' \in \mathcal{Q}$ with $Q = \gamma(\widehat{Q})$, $\widehat{Q} = [a,b]$, $Q' = \gamma(\widehat{Q}')$, $\widehat{Q}' = [a',b']$ for some $a,b,a',b' \in [0,1]$. Then, elementary integral transformations show that

$$\begin{aligned}&\int_Q\int_{Q'}\phi(y)\psi(x)G(x,y)\,\mathrm{d}y\,\mathrm{d}x \\ &= \int_{\widehat{Q}}\int_{\widehat{Q}'}\phi\big(\gamma(t)\big)\psi\big(\gamma(s)\big)G\big(\gamma(s),\gamma(t)\big)|\gamma'(t)|\,|\gamma'(s)|\,\mathrm{d}t\,\mathrm{d}s \\ &= \int_0^1\int_0^1 \phi\big(\gamma_{Q'}(\tau)\big)\psi\big(\gamma_Q(\sigma)\big)G\big(\gamma_Q(\sigma),\gamma_{Q'}(\tau)\big)|\gamma_{Q'}'(\tau)|\,|\gamma_Q'(\sigma)|\,\mathrm{d}\tau\,\mathrm{d}\sigma,\end{aligned}$$

where $\gamma_Q(\sigma) := \gamma\big(a+\sigma(b-a)\big)$ and $\gamma_{Q'}(\tau) := \gamma\big(a'+\tau(b'-a')\big)$. We abbreviate $\widetilde{\phi}(\tau) := \phi\big(\gamma_{Q'}(\tau)\big)|\gamma_{Q'}'(\tau)|$ and $\widetilde{\psi}(\sigma) := \psi\big(\gamma_Q(\sigma)\big)|\gamma_Q'(\sigma)|$, which gives that

$$\int_Q\int_{Q'}\phi(y)\psi(x)G(x,y)\,\mathrm{d}y\,\mathrm{d}x = \int_0^1\int_0^1 \widetilde{\phi}(\tau)\widetilde{\psi}(\sigma)G\big(\gamma_Q(\sigma),\gamma_{Q'}(\tau)\big)\,\mathrm{d}\tau\,\mathrm{d}\sigma.$$

Now, we distinguish three cases. For the sake of better readability, we will drop the differential terms as $\mathrm{d}\sigma$ and $\mathrm{d}\tau$.

Case 1 (no intersection): We assume that $Q \cap Q' = \emptyset$. In this case, the last term is of the form $\int_0^1\int_0^1 f(\sigma,\tau)\,\mathrm{d}\tau\,\mathrm{d}\sigma$ with some (at least) continuous function f on $[0,1]\times[0,1]$. Similarly as before, one now chooses quadrature points $(\sigma_k)_{k=1}^{n_1}$ and $(\tau_k)_{k=1}^{n_2}$ and replaces f in the integral by $f_1 \otimes f_2 := (\sigma,\tau) \mapsto f_1(\sigma)f_2(\tau)$ where f_1

and f_2 are polynomials of order $n_1 - 1$ and $n_2 - 1$, respectively, and $f_1 \otimes f_2$ interpolates f at all $n_1 \cdot n_2$ points $(\sigma_{k_1}, \tau_{k_2})$. Choosing both $(\sigma_k)_{k=1}^{n_1}$ and $(\tau_k)_{k=1}^{n_2}$ as the Gaussian quadrature points and letting again $n_1 \to \infty$ and $n_2 \to \infty$ results in convergence of the approximation to the exact integral, where the convergence is even exponential with respect to the number of points if the integrand is smooth [13, Section 5.3].

Case 2 (identical elements): We assume that $Q = Q'$, which also implies that $a = a'$ and $b = b'$. We split the integral into two summands, use Fubini's theorem as well as the reflection $(\sigma, \tau) \mapsto (\tau, \sigma)$ for the second one, and use the *Duffy transformation* $(\sigma, \tau) \mapsto (\sigma, \sigma\tau)$ with Jacobi determinant σ (see Figure 1 for a visualization)

$$
\begin{aligned}
&\int_0^1 \int_0^1 \widetilde{\phi}(\tau)\widetilde{\psi}(\sigma) G\big(\gamma_Q(\sigma), \gamma_{Q'}(\tau)\big) \\
&= \int_0^1 \int_0^\sigma \widetilde{\phi}(\tau)\widetilde{\psi}(\sigma) G\big(\gamma_Q(\sigma), \gamma_Q(\tau)\big) + \int_0^1 \int_\sigma^1 \widetilde{\phi}(\tau)\widetilde{\psi}(\sigma) G\big(\gamma_Q(\sigma), \gamma_Q(\tau)\big) \\
&= \int_0^1 \int_0^\sigma \widetilde{\phi}(\tau)\widetilde{\psi}(\sigma) G\big(\gamma_Q(\sigma), \gamma_Q(\tau)\big) + \int_0^1 \int_0^\sigma \widetilde{\phi}(\sigma)\widetilde{\psi}(\tau) G\big(\gamma_Q(\tau), \gamma_Q(\sigma)\big) \\
&= \int_0^1 \int_0^1 \Big(\widetilde{\phi}(\sigma\tau)\widetilde{\psi}(\sigma) G\big(\gamma_Q(\sigma), \gamma_Q(\sigma\tau)\big) + \widetilde{\phi}(\sigma)\widetilde{\psi}(\sigma\tau) G\big(\gamma_Q(\sigma\tau), \gamma_Q(\sigma)\big)\Big)\sigma
\end{aligned}
$$

where we omitted the differentials $\mathrm{d}\sigma$ and $\mathrm{d}\tau$ for the sake of readability. Recall that $G(x,y) = -\frac{1}{2\pi}\log|x-y|$. Thus, we further obtain (with the transformation $(\sigma, \tau) \mapsto (\sigma, 1-\tau)$ for the last integral) that

$$
\begin{aligned}
&= -\frac{1}{2\pi} \int_0^1 \int_0^1 \big(\widetilde{\phi}(\sigma\tau)\widetilde{\psi}(\sigma) + \widetilde{\phi}(\sigma)\widetilde{\psi}(\sigma\tau)\big) \log\left(\frac{\big|\gamma_Q(\sigma) - \gamma_Q(\sigma\tau)\big|}{\sigma - \sigma\tau}(\sigma - \sigma\tau)\right)\sigma \\
&= -\frac{1}{2\pi} \int_0^1 \int_0^1 \big(\widetilde{\phi}(\sigma\tau)\widetilde{\psi}(\sigma) + \widetilde{\phi}(\sigma)\widetilde{\psi}(\sigma\tau)\big) \log\left(\frac{\big|\gamma_Q(\sigma) - \gamma_Q(\sigma\tau)\big|}{\sigma - \sigma\tau}\right)\sigma \\
&\quad - \frac{1}{2\pi} \int_0^1 \int_0^1 \big(\widetilde{\phi}(\sigma\tau)\widetilde{\psi}(\sigma) + \widetilde{\phi}(\sigma)\widetilde{\psi}(\sigma\tau)\big) \log(\sigma)\sigma \\
&\quad - \frac{1}{2\pi} \int_0^1 \int_0^1 \big(\widetilde{\phi}(\sigma(1-\tau))\widetilde{\psi}(\sigma) + \widetilde{\phi}(\sigma)\widetilde{\psi}(\sigma(1-\tau))\big) \log(\tau)\sigma.
\end{aligned}
$$

With the fundamental theorem of calculus and the fact that γ is (at least) piecewise continuously differentiable with $|\gamma'| > 0$, it is easy to see that $(\sigma, \tau) \mapsto \big|\gamma_Q(\sigma) - \gamma_Q(\sigma\tau)\big|/(\sigma - \sigma\tau)$ is (at least) continuous and (uniformly) larger than 0. Thus, one can use standard quadrature as in Case 1 to approximate the first integral. Note that the term $\log(\sigma)\sigma$ is continuous, but not even C^1. The corresponding integral has the form $\int_0^1 \int_0^1 f(\sigma, \tau) \log(\sigma) \,\mathrm{d}\sigma \,\mathrm{d}\tau$ with some (at least) continuous function f on $[0,1] \times [0,1]$. Replacing f as before and choosing Gaussian

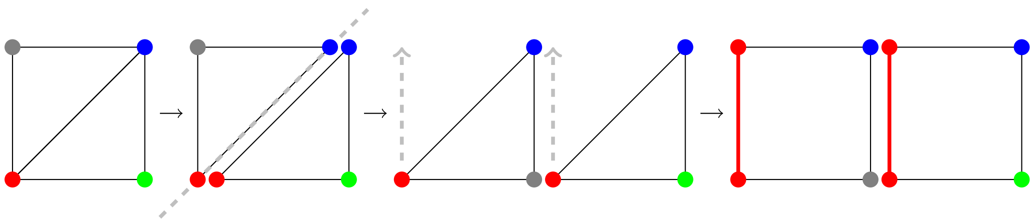


Figure 1: The integral transformations from Section 2 (the reflection $(\sigma,\tau)\mapsto(\tau,\sigma)$ for the upper triangle and the Duffy transformation $(\sigma,\tau)\mapsto(\sigma,\sigma\tau)$ for both triangles) are visualized.

quadrature points $(\sigma_k)_{k=1}^{n_1}$ with *weight function* $\log(\sigma)$ in σ-direction (see [11, Chapter 6]) and standard Gaussian quadrature points $(\tau_k)_{k=1}^{n_2}$ in τ-direction, the same convergence results apply. One can proceed similarly for the third integral.

Case 3 (common node): We assume that $Q\cap Q'$ contains only one point. Without loss of generality, we further assume that the singularity is at $(\sigma,\tau)=(0,1)$, i.e., $a=b'$ or $a=0\wedge b'=1$. We rotate the integration domain by $\pi/2$, i.e., $(\sigma,\tau)\mapsto(\tau,1-\sigma)$, which transforms the singularity to $(\sigma,\tau)=(0,0)$, and then employ the same transformations as in Case 2 (see also Figure 1)

$$\begin{aligned}
&\int_0^1\int_0^1 \widetilde{\phi}(\tau)\widetilde{\psi}(\sigma)G\big(\gamma_Q(\sigma),\gamma_{Q'}(\tau)\big)\,d\tau\,d\sigma\\
&\quad=\int_0^1\int_0^1 \widetilde{\phi}(1-\sigma)\widetilde{\psi}(\tau)G\big(\gamma_Q(\tau),\gamma_{Q'}(1-\sigma)\big)\,d\tau\,d\sigma\\
&\quad=\int_0^1\int_0^1 \Big(\widetilde{\phi}(1-\sigma)\widetilde{\psi}(\sigma\tau)G\big(\gamma_Q(\sigma\tau),\gamma_{Q'}(1-\sigma)\big)\\
&\qquad+\widetilde{\phi}(1-\sigma\tau)\widetilde{\psi}(\sigma)G\big(\gamma_Q(\sigma),\gamma_{Q'}(1-\sigma\tau)\big)\Big)\,d\tau\,d\sigma.
\end{aligned}$$

Recall that $G(x,y)=-\frac{1}{2\pi}\log|x-y|$. Thus, we further obtain that

$$\begin{aligned}
&=-\frac{1}{2\pi}\int_0^1\int_0^1 \widetilde{\phi}(1-\sigma)\widetilde{\psi}(\sigma\tau)\log\left(\frac{\big|\gamma_Q(\sigma\tau)-\gamma_{Q'}(1-\sigma)\big|}{\sigma}\,\sigma\right)\sigma\,d\tau\,d\sigma\\
&\quad-\frac{1}{2\pi}\int_0^1\int_0^1 \widetilde{\phi}(1-\sigma\tau)\widetilde{\psi}(\sigma)\log\left(\frac{\big|\gamma_Q(\sigma)-\gamma_{Q'}(1-\sigma\tau)\big|}{\sigma}\,\sigma\right)\sigma\,d\tau\,d\sigma
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{2\pi}\int_0^1\int_0^1 \widetilde{\phi}(1-\sigma)\widetilde{\psi}(\sigma\tau)\log\left(\frac{\left|\gamma_Q(\sigma\tau)-\gamma_{Q'}(1-\sigma)\right|}{\sigma}\right)\sigma\,\mathrm{d}\tau\,\mathrm{d}\sigma \\
&\quad -\frac{1}{2\pi}\int_0^1\int_0^1 \widetilde{\phi}(1-\sigma\tau)\widetilde{\psi}(\sigma)\log\left(\frac{\left|\gamma_Q(\sigma)-\gamma_{Q'}(1-\sigma\tau)\right|}{\sigma}\right)\sigma\,\mathrm{d}\tau\,\mathrm{d}\sigma \\
&\quad -\frac{1}{2\pi}\int_0^1\int_0^1 \big(\widetilde{\phi}(1-\sigma)\widetilde{\psi}(\sigma\tau)+\widetilde{\phi}(1-\sigma\tau)\widetilde{\psi}(\sigma)\big)\log(\sigma)\sigma\,\mathrm{d}\tau\,\mathrm{d}\sigma.
\end{aligned}
$$

With the fundamental theorem of calculus and the fact that γ is (at least) piecewise continuously differentiable with $|\gamma'|>0$, it is easy to see that $(\sigma,\tau)\mapsto \left|\gamma_Q(\sigma\tau)-\gamma_{Q'}(1-\sigma)\right|/\sigma$ and $(\sigma,\tau)\mapsto\left|\gamma_Q(\sigma)-\gamma_{Q'}(1-\sigma\tau)\right|/\sigma$ are (at least) continuous and (uniformly) larger than 0. Note that the term $\log(\sigma)\sigma$ is continuous, but not even C^1. Hence, we use the same quadrature as for the second and third integral in Case 2.

3 Collocation BEM

Given some basis $(\Phi_i)_{i=1}^N$ of $\mathcal{X}$, the collocation BEM approximation $\Phi=\sum_{i=1}^N c_i\Phi_i$ of (4) can be computed by solving the linear system

$$
\big((\mathfrak{V}\Phi_j)(x_i)\big)_{i,j=1}^N\cdot\big(c_i\big)_{i=1}^N=\big(g(x_i)\big)_{i=1}^N.
$$

We show how to compute $(\mathfrak{V}\phi)(x)$ for (at least) $\mathcal{Q}$-piecewise continuously differentiable path γ and (at least) $\mathcal{Q}$-piecewise continuous functions ϕ,ψ on the boundary.

Let $s\in[0,1]$ with $x=\gamma(s)$. Moreover, let $Q\in\mathcal{Q}$ with $x\in Q$ and $Q=\gamma(\widehat{Q})$, $\widehat{Q}=[a,b]$ for some $a,b\in[0,1]$. We start with

$$
(\mathfrak{V}\phi)(x)=\int_\Gamma\phi(y)G(x,y)\,\mathrm{d}y=\sum_{\widehat{Q}'\in\widehat{\mathcal{Q}}}\int_{\widehat{Q}'}\phi\big(\gamma(t)\big)G\big(\gamma(s),\gamma(t)\big)\left|\gamma'(t)\right|\mathrm{d}t.
$$

For all $\widehat{Q}'\in\widehat{\mathcal{Q}}$ with $\widehat{Q}'\neq\widehat{Q}$, the integrands are (at least) continuous and we can use standard quadrature as explained in Section 2 s after transforming the integration domain to the unit square. Recall that $G(x,y)=-\frac{1}{2\pi}\log|x-y|$. With the abbreviation $\overline{\phi}(t):=\phi\big(\gamma(t)\big)\left|\gamma'(t)\right|$, it remains to consider

$$
\int_{\widehat{Q}'}\overline{\phi}(t)G\big(\gamma(s),\gamma(t)\big)\,\mathrm{d}t=\int_a^s\overline{\phi}(t)G\big(\gamma(s),\gamma(t)\big)\,\mathrm{d}t+\int_s^b\overline{\phi}(t)G\big(\gamma(s),\gamma(t)\big)\,\mathrm{d}t
$$

$$
\begin{aligned}
&= (s-a)\int_0^1 \overline{\phi}\big(a+\tau(s-a)\big)\,G\Big(\gamma(s),\gamma\big(a+\tau(s-a)\big)\Big)\,\mathrm{d}\tau \\
&\quad + (b-s)\int_0^1 \overline{\phi}\big(s+\tau(b-s)\big)\,G\Big(\gamma(s),\gamma\big(s+\tau(b-s)\big)\Big)\,\mathrm{d}\tau \\
&= -\frac{1}{2\pi}(s-a)\int_0^1 \overline{\phi}\big(a+\tau(s-a)\big)\log\left(\frac{\big|\gamma(s)-\gamma\big(a+\tau(s-a)\big)\big|}{\tau}\right)\mathrm{d}\tau \\
&\quad - \frac{1}{2\pi}(s-a)\int_0^1 \overline{\phi}\big(a+\tau(s-a)\big)\log(\tau)\,\mathrm{d}\tau \\
&\quad - \frac{1}{2\pi}(b-s)\int_0^1 \overline{\phi}\big(s+\tau(b-s)\big)\log\left(\frac{\big|\gamma(s)-\gamma\big(s+\tau(b-s)\big)\big|}{\tau}\right)\mathrm{d}\tau \\
&\quad - \frac{1}{2\pi}(b-s)\int_0^1 \overline{\phi}\big(s+\tau(b-s)\big)\log(\tau)\,\mathrm{d}\tau.
\end{aligned}
$$

With the fundamental theorem of calculus and the fact that γ is (at least) piecewise continuously differentiable with $|\gamma'| > 0$, it is easy to see that $\tau \mapsto \big|\gamma(s)-\gamma\big(a+\tau(s-a)\big)\big|/\tau$ and $\tau \mapsto |\gamma(s)-\gamma\big(s+\tau(b-s)\big)\big|/\tau$ are (at least) continuous and (uniformly) larger than 0. Thus, we can use quadrature with weight function 1 and $\log(\tau)$ as in Section 2 to approximate the final integrals.

4 Faermann estimator

The computation of the error indicators $\eta(Q)$, $Q \in \mathcal{Q}$, of (5) can be realized by means of Section 2–3. Let $Q' \in \mathcal{Q}\setminus\{Q\}$ with $Q \cap Q' \neq \emptyset$. Recall the abbreviation of the residual $r := f - \mathfrak{V}\Phi$. Then,

$$
\begin{aligned}
&\int_{Q\cup Q'}\int_{Q\cup Q'} \frac{\big|r(x)-r(y)\big|^2}{|x-y|^2}\,\mathrm{d}y\,\mathrm{d}x \\
&\quad = \int_Q\int_Q \frac{\big|r(x)-r(y)\big|^2}{|x-y|^2}\,\mathrm{d}y\,\mathrm{d}x + \int_{Q'}\int_{Q'} \frac{\big|r(x)-r(y)\big|^2}{|x-y|^2}\,\mathrm{d}y\,\mathrm{d}x \\
&\qquad + 2\int_Q\int_{Q'} \frac{\big|r(x)-r(y)\big|^2}{|x-y|^2}\,\mathrm{d}y\,\mathrm{d}x.
\end{aligned}
$$

Given the evaluation procedures of Section 3, the first two terms can be approximated exactly as in Case 2 of Section 2, where one can additionally exploit the symmetry of the integrand. The remaining integral can be approximated exactly as in Case 3 of Section 2.

5 Acknowledgements

The author thankfully acknowledges support by the Austrian Science Fund (FWF) through the Erwin Schrödinger Fellowship *Optimal adaptivity for space-time methods* (grant J4379).

References

[1] Francesco Calabrò, Antonella Falini, Maria Lucia Sampoli, and Alessandra Sestini, *Efficient quadrature rules based on spline quasi-interpolation for application to IGA-BEMs*, J. Comput. Appl. Math. **338** (2018), 153–167.

[2] J. Austin Cottrell, Thomas J. R. Hughes, and Yuri Bazilevs, *Isogeometric analysis: toward integration of CAD and FEA*, John Wiley & Sons, New York, 2009.

[3] Birgit Faermann, *Localization of the Aronszajn-Slobodeckij norm and application to adaptive boundary elements methods. Part I. The two-dimensional case*, IMA J. Numer. Anal. **20** (2000), no. 2, 203–234.

[4] ______, *Localization of the Aronszajn-Slobodeckij norm and application to adaptive boundary element methods. Part II. The three-dimensional case*, Numer. Math. **92** (2002), no. 3, 467–499.

[5] Michael Feischl, Gregor Gantner, Alexander Haberl, and Dirk Praetorius, *Adaptive 2D IGA boundary element methods*, Eng. Anal. Bound. Elem. **62** (2016), 141–153.

[6] Michael Feischl, Gregor Gantner, and Dirk Praetorius, *Reliable and efficient a posteriori error estimation for adaptive IGA boundary element methods for weakly-singular integral equations*, Comput. Methods Appl. Mech. Engrg. **290** (2015), 362–386.

[7] Thomas Führer, Gregor Gantner, Dirk Praetorius, and Stefan Schimanko, *Optimal additive Schwarz preconditioning for adaptive 2d IGA boundary element methods*, Comput. Methods Appl. Mech. Engrg. **351** (2019), 571–598.

[8] Gregor Gantner, *Optimal adaptivity for splines in finite and boundary element methods*, Ph.D. thesis, TU Wien, 2017.

[9] Gregor Gantner, Dirk Praetorius, and Stefan Schimanko, *Adaptive isogeometric boundary element methods with local smoothness control*, Math. Models Methods Appl. Sci. **30** (2020), 261–307.

[10] William McLean, *Strongly elliptic systems and boundary integral equations*, Cambridge University Press, Cambridge, 2000.

[11] Robert Plato, *Numerische Mathematik kompakt*, Vieweg+Teubner, Wiesbaden, 2000.

[12] Costas Politis, Alexandros I. Ginnis, Panagiotis D. Kaklis, Kostas Belibassakis, and Christian Feurer, *An isogeometric BEM for exterior potential-flow problems in the plane*, 2009 SIAM/ACM Joint Conference on Geometric and Physical Modeling, 2009, pp. 349–354.

[13] Stefan A. Sauter and Christoph Schwab, *Boundary element methods*, Springer, Berlin, 2011.

[14] Olaf Steinbach, *Numerical approximation methods for elliptic boundary value problems*, Springer, New York, 2008.

Authors' address:
University of Amsterdam, Korteweg-de Vries Institute for Mathematics, Postbus 94248, NL-1090 GE Amsterdam, The Netherlands
email g.gantner@uva.nl

Internat. Math. Nachrichten
Nr. 247 (2021), 11–18

Abel Prize 2021

Norwegian Academy of Science and Letters
Oslo

The Norwegian Academy of Science and Letters has awarded the Abel Prize for 2021 to László Lovász of Alfréd Rényi Institute of Mathematics (ELKH, MTA Institute of Excellence) and Eötvös Loránd University in Budapest, Hungary, and Avi Wigderson of the Institute for Advanced Study, Princeton, USA "for their foundational contributions to theoretical computer science and discrete mathematics, and their leading role in shaping them into central fields of modern mathematics."

The Abel Prize Laureates 2021 László Lovász and Avi Wigderson © Hungarian Academy of Sciences / Institute for Advanced Study, Princeton, NJ USA

1 Citation

Theoretical computer science (TCS) is the study of the power and limitations of computing. Its roots go back to the foundational works of Kurt Gödel, Alonzo Church, Alan Turing, and John von Neumann, leading to the development of real physical computers. TCS contains two complementary subdisciplines: algorithm

ISSN 0020-7926

design, which develops efficient methods for a multitude of computational problems, and computational complexity, which shows inherent limitations on the efficiency of algorithms. The notion of polynomial-time algorithms put forward in the 1960s by Alan Cobham, Jack Edmonds, and others and the famous P$\neq$NP conjecture of Stephen Cook, Leonid Levin, and Richard Karp had strong impact on the field and on the work of Lovász and Wigderson.

Apart from its tremendous impact on broader computer science and practice, TCS provides the foundations of cryptography and is now having growing influence on several other sciences, leading to new insights therein by "employing a computational lens." Discrete structures such as graphs, strings, permutations are central to TCS, and naturally discrete mathematics and TCS have been closely allied fields. While both these fields have benefited immensely from more traditional areas of mathematics, there has been growing influence in the reverse direction as well. Applications, concepts, and techniques from TCS have motivated new challenges, opened new directions of research, and solved important open problems in pure and applied mathematics.

László Lovász and Avi Wigderson have been leading forces in these developments over the last decades. Their work interlaces in many ways, and, in particular, they have both made fundamental contributions to understanding randomness in computation and in exploring the boundaries of efficient computation.

Along with Arjen Lenstra and Hendrik Lenstra, László Lovász developed the LLL lattice reduction algorithm. Given a high-dimensional integer lattice (grid), this algorithm finds a nice, nearly orthogonal basis for it. In addition to several applications, such as an algorithm to factorize rational polynomials, the LLL algorithm is a favorite tool of cryptanalysts, successfully breaking several proposed cryptosystems. Surprisingly, the analysis of the LLL algorithm is also used to design and guarantee the security of newer, lattice-based cryptosystems that seem to withstand attacks even by quantum computers. For some exotic cryptographic primitives, such as homomorphic encryption, the only constructions known are via these lattice-based cryptosystems.

The LLL algorithm is only one among many of Lovász's visionary contributions. He proved the Local Lemma, a unique tool to show existence of combinatorial objects whose existence is rare, as opposed to the standard probabilistic method used when objects exist in abundance. Along with Martin Grötschel and Lex Schrijver, he showed how to efficiently solve semidefinite programs, leading to a revolution in algorithm design. He contributed to the theory of random walks with applications to Euclidean isoperimetric problems and approximate volume computations of high-dimensional bodies. His paper with Uriel Feige, Shafi Goldwasser, Shmuel Safra, and Mario Szegedy on probabilistically checkable proofs gave an early version of the PCP Theorem, an immensely influential result showing that the correctness of mathematical proofs can be verified probabilistically, with high confidence, by reading only a small number of symbols! In addition, he also solved long-standing problems such as the perfect graph conjecture, the Kneser conjecture, determining the Shannon capacity of the pentagon graph, and, in recent years, developed the theory of graph limits (in joint work with Christian Borgs, Jennifer Chayes, Lex Schrijver, Vera Sós, Balázs Szegedy, and Katalin Vesztergombi). This work ties together elements of extremal graph theory, probability theory, and statistical physics.

Avi Wigderson has made broad and profound contributions to all aspects of computational complexity, especially the role of randomness in computation. A randomized algorithm is one that flips coins to compute a solution that is correct with high probability. Over decades, researchers discovered deterministic algorithms for many problems for which only a randomized algorithm was known before. The deterministic algorithm for primality testing, by Agrawal, Kayal, and Saxena, is a striking example of such a derandomized algorithm. These derandomization results raise the question of whether randomness is ever really essential. In works with László Babai, Lance Fortnow, Noam Nisan, and Russell Impagliazzo, Wigderson demonstrated that the answer is likely to be in the negative. Formally, they showed a computational conjecture, similar in spirit to the P$\neq$NP conjecture, implies that P$=$BPP. This means that every randomized algorithm can be derandomized and turned into a deterministic one with comparable efficiency; moreover, the derandomization is generic and universal, without depending on the internal details of the randomized algorithm.

Another way to look at this work is as a tradeoff between hardness versus randomness: if there exists a hard enough problem, then randomness can be simulated by efficient deterministic algorithms. Wigderson's subsequent work with Impagliazzo and Valentine Kabanets proves a converse: efficient deterministic algorithms even for specific problems with known randomized algorithms would imply that there must exist such a hard problem.

This work is intimately tied with constructions of pseudorandom (random looking) objects. Wigderson's works have constructed pseudorandom generators that turn a few truly random bits into many pseudorandom bits, extractors that extract nearly perfect random bits from an imperfect source of randomness, Ramsey graphs and expander graphs that are sparse and still have high connectivity. With Omer Reingold and Salil Vadhan, he introduced the zig-zag graph product, giving an elementary method to build expander graphs, and inspiring the combinatorial proof of the PCP Theorem by Irit Dinur and a memory efficient algorithm for the graph connectivity problem by Reingold. The latter gives a method to navigate through a large maze while remembering the identity of only a constant number of intersection points in the maze!

Wigderson's other contributions include zero-knowledge proofs that provide proofs for claims without revealing any extra information besides the claims' validity, and lower bounds on the efficiency of communication protocols, circuits, and formal proof systems.

Thanks to the leadership of Lovász and Wigderson, discrete mathematics and the relatively young field of theoretical computer science are now established as central areas of modern mathematics.

2 Biography of László Lovász

A mathematical star since he was a teenager, László Lovász has more than delivered on his early promise, becoming one of the most prominent mathematicians of the last half century. His work has established connections between discrete mathematics and computer science, helping to provide theoretical foundations, as well as design practical applications, for these two large and increasingly important areas of scientific study. He has also served his community as a prolific writer of books, noted for their clarity and accessibility, as an inspirational lecturer, and as a leader, spending a term as president of the International Mathematical Union (2007-2010).

Born in 1948 in Budapest, Lovász was part of a golden generation of young Hungarian mathematicians, nurtured by the country's unique school mathematics culture. He was in the first group of an experiment in which gifted students at a Budapest high school were given specialist math classes. (One of his classmates was Katalin Vesztergombi, whom he later married.) Lovász excelled, winning gold medals in the 1964, 1965, and 1966 International Mathematics Olympiads, on the latter two occasions with perfect scores. He also won a primetime Hungarian TV show in which students were placed in glass cages and asked to solve math problems.

Perhaps the most important encounter in his teenage years, however, was with his mathematical hero, Paul Erdős, the nomadic and famously sociable Hungarian mathematician. Erdős was an insatiable sharer of problems and inspired Lovász to work in "Hungarian-style combinatorics", essentially concerned with properties of graphs. Not only did this set up an initial research direction, but it also paved the way for Lovász's style of how to do math: openly and collaboratively.

Lovász attended Eötvös Loránd University in Budapest. He was awarded a PhD (or rather, the Hungarian equivalent, the CSc) at age twenty-two in 1970, by which time he had already lectured at international conferences and had fifteen papers published. Due to a quirk of the Hungarian system, he only graduated in 1971, a year after he got his PhD.

Combinatorics is the math of patterns and counting patterns. Graph theory is the math of connections, such as in a network. Both come under the umbrella of "discrete" math, since the objects of study have distinct values, rather than varying smoothly like, say, a point moving along a curve. Erdős liked to study these areas for purely intellectual pleasure, with no concern for their usefulness in the real world. Lovász, on the other hand, became a leader of a new generation of mathematicians who realized that discrete math had, in computer science, a thrilling new area of application.

In the 1970s, for example, graph theory became one of the first areas of pure mathematics able to illuminate the new field of computational complexity. Indeed, one of the major impacts of Lovász's work has been to establish ways in which discrete math can address fundamental theoretical questions in computer science.

Among his contributions to the foundational underpinning of computer science are powerful algorithms with wide-ranging applications. One of these, the LLL algorithm, named after Lovász and the brothers Arjen and Hendrik Lenstra, represented a conceptual breakthrough in the understanding of lattices, a basic geometrical object, and which has had remarkable applications in areas including number theory, cryptography, and mobile computing. Currently, the only known encryption systems that can withstand an attack by a quantum computer are based on lattices and use the LLL algorithm.

During the 1970s and 1980s, Lovász was based in Hungary, first at Eötvös Loránd University and then at József Attila University in Szeged, where he became chair of Geometry in 1978. He returned to Eötvös Loránd in 1982 to be chair of Computer Science. In those early decades he solved important and far-reaching problems in many areas of discrete mathematics. One of his first major results, in 1972, was to resolve the "perfect graph conjecture", a long-standing open problem in graph theory. In 1978 he settled Kneser's conjecture, again in graph theory, but this time surprising his colleagues by using a proof from algebraic topology, a completely different area. In 1979 he solved a classical problem in information theory, determining the "Shannon capacity" of the pentagon graph.

A major theme of Lovász's work in both combinatorics and algorithm design is the investigation of probabilistic methods. The discovery in this area for which he is best known is the Lovász Local Lemma, an important and frequently used tool in probabilistic combinatorics used to establish the existence of rare objects, as opposed to the more standard tools used when objects are more abundant. Lovász also contributed to an early, influential paper on probabilistically checkable proofs (PCP), which grew into one of the most important areas of computational complexity.

In 1993, Lovász was appointed William K. Lanman Professor of Computer Science and Mathematics at Yale University. In 1999, he left academia to take up a position as a Senior Researcher at Microsoft, before returning in 2006 to Eötvös Loránd University, where he is currently a professor.

Lovász has traveled widely. He has held visiting positions at the universities of Vanderbilt in Nashville, Tennessee (1972-1973), Waterloo (1978-1979), Bonn (1984- 1985), Chicago (1985), Cornell (1985), and Princeton (1989-1993), as well as spending a year at the Institute for Advanced Study in Princeton (2011-2012). Called "Laci" by friends and colleagues, he is known for his modesty, generosity, and openness. These qualities have led to positions on the executive committee of the International Mathematical Union (including as president), and at the Hungarian Academy of Sciences (where he was president from 2014 to 2020).

Lovász has won many awards, including the 1999 Wolf Prize, the 1999 Knuth Prize, the 2001 Gödel Prize, and the 2010 Kyoto Prize.

He has four children with Katalin Vesztergombi, a mathematician who is also one of his frequent collaborators, and seven grandchildren.

Source for quote: Simons Foundation, interview with Lásló Lovász, 2013.

3 Biography of Avi Wigderson

When Avi Wigderson began his academic career in the late 1970s, the theory of "computational complexity" – which concerns itself with the speed and efficiency of algorithms – was in its infancy. Wigderson's contribution to enlarging and deepening the field is arguably greater than that of any single other person, and what was a young subject is now an established field of both mathematics and theoretical computer science. Computational complexity has also become unexpectedly important, providing the theoretical basis for Internet security.

Wigderson was born in Haifa, Israel, in 1956. He entered Technion, the Israeli Institute of Technology, in 1977, and graduated with a BSc in computer science in 1980. He moved to Princeton for his graduate studies, receiving his PhD in 1983 for the thesis Studies in Combinatorial Complexity, for which Richard Lipton was his advisor. In 1986, Wigderson returned to Israel to take up a position at the Hebrew University in Jerusalem. He was given tenure the following year and made full professor in 1991.

In the 1970s, computer theoreticians framed certain fundamental ideas about the nature of computation, notably the notions of P and NP. P is the set of problems that computers can solve easily, say, in a few seconds, whereas NP also contains problems that computers find hard to solve, meaning that the known methods can only find the answer in, say, millions of years. The question of whether all these hard problems can be reduced to easy ones, that is, whether or not P=NP, is the foundational question of computational complexity. Indeed, it is now considered one of the most important unsolved questions in all of mathematics.

Wigderson made stunning advances in this area by investigating the role of randomness in aiding computation. Some hard problems can be made easy using algorithms in which the computer flips coins during the computation. If an algorithm relies on coin flipping, however, there is always a chance that an error can creep into the solution. Wigderson, first together with Noam Nisan and later with Russell Impagliazzo, showed that for any fast algorithm that can solve a hard problem using coin flipping, there exists an almost-as-fast algorithm that does not use coin flipping, provided certain conditions are met.

Wigderson has conducted research into every major open problem in complexity theory. In many ways, the field has grown around him, not only because of his breadth of interests, but also because of his approachable personality and enthusiasm for collaborations. He has coauthored papers with well over 100 people and has mentored a large number of young complexity theorists.

In 1999, Wigderson joined the Institute for Advanced Study (IAS) in Princeton, where he has been ever since. At an event to celebrate Wigderson's sixtieth birthday in 2016, IAS director Robbert Dijkgraaf said that he had launched a golden age of theoretical computer science at the institute.

Wigderson is known for his ability to see links between apparently unrelated areas. He has deepened the connections between mathematics and computer science. One example is the "zig-zag graph product", which he developed with Omer Reingold and Salil Vadhan, which links group theory, graph theory, and complexity theory and has surprising applications, such as how best to get out of a maze.

The most important present-day application of complexity theory is to cryptography, which is used to secure information on the Internet, such as credit card numbers and passwords. People who design cryptosystems, for example, must make sure that the task of decoding their system is an NP problem, that is, one that would take computers millions of years to achieve. Early in his career, Wigderson made fundamental contributions to a new concept in cryptography, the zero-knowledge proof, which more than thirty years later is now being used in blockchain technology. In a zero-knowledge proof, two people must prove a claim without revealing any knowledge beyond the validity of that claim, such as the example of the two millionaires who want to prove who is richer without either of them letting on how much money they have. Wigderson, together with Oded Goldreich and Silvio Micali, showed that zero-knowledge proofs can be used to prove, in secret, any public result about secret data. Just say, for example, that you want to prove to someone that you have proved a mathematical theorem, but you don't want to reveal any details of how you did it, a zero-knowledge proof will allow you to do this.

In 1994, Wigderson won the Rolf Nevanlinna Prize for computer science, which is awarded by the International Mathematical Union every four years. Among his many other prizes are the 2009 Gödel Prize and the 2019 Knuth Prize.

Wigderson is married to Edna, whom he met at the Technion, and who works in the computer department of the Institute for Advanced Study. They have three children and two grandchildren.

Source for quote: Heidelberg Laureate Foundation Portraits, interview with Avi Widgerson, 2017.

All material in this article is taken from the Abel Prize website https://www.abelprize.no/c76389/seksjon/vis.html?tid=76390&strukt_tid=76389. It is printed with friendly permission by the Norwegian Academy of Science and Letters/The Able Prize (represented by Head of Communications Marina Tofting). A similar version of this article has appeared in the June/July 2021 issue of the Notices of the American Mathematical Society *("Lovász and Wigderson Awarded 2021 Abel Prize", Notices of the American Mathematical Society, Volume 68-6, June/July 2021, 1034-1037 © American Mathematical Society).*

Internat. Math. Nachrichten
Nr. 247 (2021), 19–28

Ein Brief von Eisensteins Eltern an Gauß

Franz Lemmermeyer
Gymnasium St. Gertrudis

Constantin und Helene Eisenstein, die Eltern von Gotthold Eisenstein, haben sich nach dem Tod ihres Sohnes brieflich bei Carl Friedrich Gauß bedankt. Dieser Brief ist inzwischen online[1] gestellt. Hier wollen wir eine Transkription des Briefs[2] geben sowie den mathematisch interessanten Teil der Anlage, nämlich einen bisher unveröffentlichten Beweis des quadratischen Reziprozitätsgesetzes mithilfe der Tangensfunktion.

1 Der Brief von Eisensteins Eltern an Gauß

An
S[eine]r Hochwohlgeboren
den Herrn Geheimen-Hofrath
Professor Dr Gauss
Zu Göttingen

Berlin den 12tn November 1852.

Hochwohlgeborner Herr!

Wenn gleich Herr Professor D$^{\mathrm{r}}$ Encke die Güte gehabt hat Ihnen Hochgeehrter Herr das Ableben unseres Sohnes Dr Gotthold Eisenstein zur Zeit anzuzeigen; so bewegt uns doch die Pflicht der Dankbarkeit, einige Zeilen an den Wohlthäter und Meister unseres Kindes Ehrfurchtsvoll zu richten.

[1] Siehe https://gauss.adw-goe.de/handle/gauss/2588?.

[2] Erstellt mit freundlicher Unterstützung durch Menso Folkerts, dem ich dafür herzlich danke. Ebenso herzlich danke ich Peter Ullrich für Kommentare und Korrekturen.

ISSN 0020-7926

Mit traurigem Herzen wiederholen wir Ihnen, daß unser vielgeliebtes letztes Kind (dem Fünf voraus gegangen sind) Ferdinand Gotthold Maximilian Eisenstein Dr der Phylosophie etc geboren zu Berlin am 16tn April 1823, am 11tn October 1852 früh 1/4 vor 6 Uhr in Folge von Nervenabzehrung gestorben ist. Unser Schmerz ist groß! Doch hat der Allmächtige wohlgethan, denn er war sein ganzes Leben hindurch leidend und war es uns Eltern von der Vorsehung nicht beschieden ihm ein sorgenfreies Leben bereiten zu können, wodurch er auch von dieser Seite stets bedrängt war.

Die väterliche Theilnahme die Sie Hochgeehrter Meister an unserm Sohne persönlich und die Anerkenntniße an seinen Leistungen haben ihn oft in trüben Stunden aufgerichtet und wieder frisch an's Werk gehen lassen. Wir statten Ihnen dafür unsern herzlichsten Dank ab.

Welcher Idial Meister Sie ihm waren, läßt sich mit Worten nicht aussprechen! Ihre liebevollen Briefe die Sie an ihn gerichtet, sind in unsern Händen und sollen uns als theures Andenken von Ihnen Hochgeehrter Herr verbleiben.

Das beikommende Buch hatte für ihn keinen anderen Namen als mein Gauss, daher bitten wir Sie, dasselbe mit seinen Anmerkungen und Einlagen, wie wir es nach seinem Ableben vorgefunden haben, hochgeneigst als Andenken an ihn annehmen zu wollen. Auch erlauben wir uns, Ihnen seine letzte Academische Abhandlung und Antrittsrede beizulegen, wie auch einige mathematische Arbeiten welche seine letzten waren, und wenige Wochen vor seinem Tode ihm aus der Feder geflossen sind. Wir haben in seinem Nachlasse sehr viele mathematische Arbeiten gefunden in welchen manche schöne Gedanken enthalten sein dürften.

Möge der Allmächtige Ewr. Hochwohlgeboren noch lange in Ihren Wirkungskreise gesund erhalten, dieses wünschend von ganzem Herzen

die Sie Hochverehrenden
Eltern des Verstorbenen

Constantin Eisenstein und
Helene Eisenstein

Neue Grünstraße N^o 13

Der Beweis des quadratischen Reziprozitätsgesetzes

Den analogen Beweis für das die quadratischen Reste in der reellen Theorie betreffende Fundamentaltheorem mit Hülfe der Kreisfunktionen will ich hier noch beifügen.

$$(\cos x + i \sin x)^q = \cos qx + i \sin qx = \sum_{m=0}^{m=q} i^m q_m \cos x^{\,q-m} \sin x^{\,m}$$

$$\begin{aligned}\operatorname{tang} qx &= \frac{\sum_{n=0}^{n=\frac{q-1}{2}} (-1)^n q_{2n+1} \cos x^{\,q-2n-1} \sin x^{\,2n+1}}{\sum_{n=0}^{n=\frac{q-1}{2}} (-1)^n q_{2n} \cos x^{\,q-2n} \sin x^{\,2n}} \\ &= \frac{\sum (-1)^n q_{2n+1} \operatorname{tang} x^{\,2n+1}}{\sum (-1)^n q_{2n} \operatorname{tang} x^{\,2n}},\end{aligned}$$

welches sich, wenn q eine ungerade Primzahl bedeutet, auf die Form bringen läßt:

$$\frac{q\phi(\operatorname{tang} x^{\,2}) + (-1)^{\frac{q-1}{2}} \operatorname{tang} x^{\,q-1}}{1 + q\psi(\operatorname{tang} x^{\,2})} \operatorname{tang} x,$$

wo ϕ und ψ ganze Funktionen von $\operatorname{tang}^2 x$ mit ganzen Coefficienten sind.
Sei p eine von q verschiedene ungerade Primzahl, dann hat man ebenso

$$\operatorname{tang} px = \operatorname{tang} x \, \frac{p - \frac{p(p-1)(p-2)}{1 \cdot 2 \cdot 3} \operatorname{tang} x^{\,2} + \ldots + (-1)^{\frac{p-1}{2}} \operatorname{tang} x^{\,p-1}}{1 - \frac{p(p-1)}{1 \cdot 2} \operatorname{tang} x^{\,2} + \ldots + (-1)^{\frac{p-1}{2}} p \operatorname{tang} x^{\,p-1}}.$$

Die Wurzeln der Gleichung

$$(-1)^{\frac{p-1}{2}} Z^{p-1} + \ldots + p = 0$$

sind dann offenbar in der Formel enthalten $\operatorname{tang} \frac{\rho\omega}{p}$, wenn ρ ein vollständiges Restsystem (mod p) mit Ausschluß der Null durchläuft, und $\omega = 2\pi$ ist. Man hat hiernach

$$\prod \operatorname{tang} \frac{\rho\omega}{p} = (-1)^{\frac{p-1}{2}} p.$$

(Die Multiplication links bezieht sich auf alle ρ.)
Die Werthe von ρ lassen sich folgendermaßen gruppieren:

$$\begin{array}{c|c} r_1 & -r_1 \\ r_2 & -r_2 \\ \vdots & \ldots \\ r_{\frac{p-1}{2}} & -r_{\frac{p-1}{2}} \end{array}$$

Hieraus schließt man

$$\prod \left(\operatorname{tang} \frac{r\omega}{p} \right)^2 = p. \tag{6}$$

r bedeutet den Inbegriff der Zahlen $r_1, r_2, \ldots, r_{\frac{p-1}{2}}$.

Die Größen $(\tang \frac{r\omega}{p})^2$ sind die Wurzeln einer Gleichung mit ganzen Coefficienten

$$(-1)^{\frac{p-1}{2}} Z^{p-1} + \ldots + p = 0,$$

also ist jede symmetrische Verbindung dieser Größen einer ganzen Zahl gleich.

Multipliciert man alle r mit q, so hat man entweder $qr \equiv r'$ oder $qr \equiv -r'$ (mod p), wo r' sich jedesmal unter den r befindet. Nach diesen beiden Fällen hat man resp.

$$\tang \frac{qr\omega}{p} = \tang \frac{r'\omega}{p} \quad \text{oder} \quad = -\tang \frac{r'\omega}{p}$$

also in beiden Fällen

$$qr \equiv r' \frac{\tang \frac{qr\omega}{p}}{\tang \frac{r'\omega}{p}} \quad (\text{mod } p),$$

und da alle r' alle r erschöpfen, so schließt man hieraus

$$q^{\frac{p-1}{2}} \prod(r) \equiv \prod(r) \prod \left\{ \frac{\tang \frac{qr\omega}{p}}{\tang \frac{r\omega}{p}} \right\} \quad (\text{mod } p),$$

$$\left(\frac{q}{p}\right) = \prod \left\{ \frac{\tang \frac{qr\omega}{p}}{\tang \frac{r\omega}{p}} \right\} \quad (\text{mod } p).$$

Aber nach dem Obigen ist

$$\frac{\tang qx}{\tang x} = \frac{q\phi(\tang x^{\,2}) + (-1)^{\frac{q-1}{2}} \tang x^{\,q-1}}{1 + q\psi(\tang x^{\,2})},$$

also

$$\prod \left\{ \frac{\tang \frac{qr\omega}{p}}{\tang \frac{r\omega}{p}} \right\} = \prod \frac{\left(q\phi(\{\tang \frac{r\omega}{p}\}^2) + (-1)^{\frac{q-1}{2}} (\tang \frac{r\omega}{p})^{q-1}\right)}{1 + q\psi(\{\tang \frac{r\omega}{p}\}^2)}$$
$$= \frac{qP + (-1)^{\frac{p-1}{2}\frac{q-1}{2}} \prod\{\tang \frac{r\omega}{p}\}^{q-1}}{1 + qQ}$$
$$= \frac{qP + (-1)^{\frac{p-1}{2}\frac{q-1}{2}} p^{\frac{q-1}{2}}}{1 + qQ} \quad \text{nach (6)}$$

also

$$\left(\frac{q}{p}\right) = \frac{qP + (-1)^{\frac{p-1}{2}\frac{q-1}{2}} p^{\frac{q-1}{2}}}{1 + qQ},$$

woraus unmittelbar das Reciprocitätsgesetz folgt.

Die Werthe von P und Q können übrigens direct hingeschrieben werden, wenn man die symmetrischen Funktionen, welche sie darstellen, nach dem Newtonschen Theorem in den Coefficienten der Gleichung ausdrückt. Sie werden hiernach Aggregate von Binomialkoefficienten.

Ähnliches gilt für das biquad. Fundamentaltheorem, nur kennt man hier noch nicht das allgemeine Gesetz der Coefficienten in den Multiplicationsformeln. Letztere Coefficienten, die ich durch A_1, A_2,... bezeichnet habe, scheinen einiger Aufmerksamkeit würdig zu sein. Ich habe bloß gefunden, daß wenn man $m = a + bi$ setzt, die in Rede stehenden Coefficienten durchaus nicht ganze Funktionen von m, sondern vielmehr Funktionen der beiden getrennten Variablen a und b sind. Einiges über die Natur dieser Coefficienten kann man aus dem Umstande schließen, daß der μte Coeff. im Zähler und der μte im Nenner gleichzeitig verschwinden müssen für alle zusammengehörigen Werthe von a und b, die $a^2 + b^2 < 4\mu + 1$ machen.

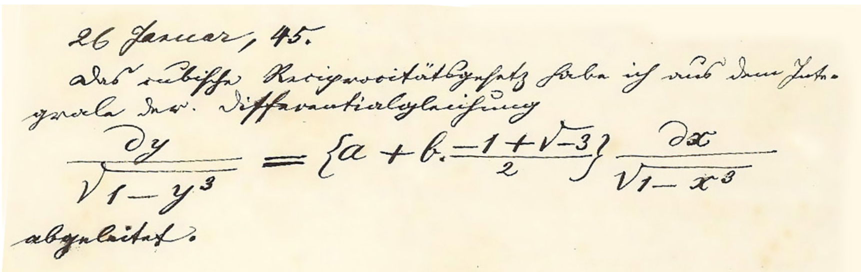
G. Eisenstein

26. Januar, 45.

Das cubische Reciprocitätsgesetz habe ich aus dem Integrale der Differentialgleichung

$$\frac{\partial y}{\sqrt{1-y^3}} = \left\{a + b\frac{-1+\sqrt{-3}}{2}\right\}\frac{\partial x}{\sqrt{1-x^3}}$$

abgeleitet.



26 Januar, 45.

Das cubische Reciprocitätsgesetz habe ich aus dem Integrale der Differentialgleichung

$$\frac{\partial y}{\sqrt{1-y^3}} = \left\{a + b.\frac{-1+\sqrt{-3}}{2}\right\}\frac{\partial x}{\sqrt{1-x^3}}$$

abgeleitet.

2 Kommentare

Neben den oben transkribierten Teilen des Briefes samt Anlagen ist nur noch der folgende kleine Ausschnitt interessant:

> *Ich darf fast gar nicht sprechen, weil ich zu sehr huste und seit 8 Tagen schon 3 mal eine Menge Blut ausgeworfen habe. Sie können deshalb immer bei mir bleiben und mir etwas erzählen.*

Ich habe schon 1 Nacht draußen in Bethanien geschlafen, aber es war zu unruhig. Mitten in der Nacht sprang ein Bierpropfen aus der Flasche etc Später jetzt bin ich zu schwach

Gotthold Eisenstein wurde Ende Juli 1852, wie Biermann auf S. 927 von Eisensteins Werken schreibt, nach einem Blutsturz für einige Tage in der Heilanstalt Bethanien untergebracht. Vermutlich stammen diese Zeilen aus dieser Zeit. An wen diese Zeilen adressiert waren, bleibt unklar.

Kommentare zum Brief

Eisensteins Eltern haben Gauß als Andenken das Exemplar seiner Disquisitiones geschenkt. Egon Ullrich, der 1925 in Graz bei Anton Rella promoviert hatte, hat das Buch in der Bibliothek des mathematischen Instituts in Gießen entdeckt; es gehörte zu den Büchern aus dem Nachlass von Eugen Netto. 1979 hat Benno Artmann diese Information an André Weil weitergegeben, der 1976 Eisensteins Werke besprochen und ein Buch über Eisensteins Zugang zu elliptischen Funktionen veröffentlicht hatte. In [23, S. 463] spekuliert er, dass Eisensteins Exemplar über den gemeinsamen Lehrer von Eisenstein und Netto, Karl Heinrich Schellbach, an Netto gekommen sein könnte, aber Peter Ullrich (siehe [21, insbes. S. 206–207] und [22]) bemerkt, dass Netto das Buch von einem wenig mathematik-affinen Buchhändler gekauft haben muss. Man wird wohl annehmen müssen, dass Ernst Schering, der den Gaußschen Nachlass geordnet und die Herausgabe der Gaußschen Werke bis zu seinem eigenen Tod geleitet hat, etwas damit zu tun gehabt hat. Warum Schering, der zahlentheoretisch durchaus bewandert war und sich ausführlich mit Beweisen des quadratischen Reziprozitätsgesetzes befasst hat, das Eisensteinsche Exemplar weggegeben haben könnte, bleibt dabei ein Rätsel. Auch was aus den mathematischen Arbeiten geworden ist, welche Eisensteins Eltern dem Brief beigelegt haben, muss offen bleiben; die Antrittsrede zu seiner Wahl in die Berliner Akademie, die Eisenstein am 1. Juli 1852 gehalten hat, ist in Band II seiner Werke abgedruckt.

Die angesprochenen Gaußschen „Anerkenntniße an seinen Leistungen“ waren für den jungen Eisenstein sicherlich eine große Motivation; auf der anderen Seite haben diese bei seinen Zeitgenossen für manchmal unverhohlenen Neid gesorgt. So schreibt Riemann am 23. Juli 1847 in einem Brief an seinen Vater ([17, S. 94]), Eisenstein habe sich „bei Gauß in Gunst zu setzen gewußt und er ist von diesem an Alexander von Humboldt empfohlen“.

Stein des Anstoßes war wohl der Brief vom 9. Juli 1845 von Gauß an von Humboldt; dort erklärt Gauß, dass er zwar Dirichlet für den Orden *Pour le mérite* vorgeschlagen habe, dass ihm diese Wahl aber schwergefallen sei:

Sollte jene Injunction aber ganz buchstäblich genommen werden müssen, nemlich unabhängig von jeder Rücksicht, also auch von der, ob *einige Aussicht sei, daß dem Vorschlage, wie ungewöhnlich er auch sei, Folge gegeben werden könne, so bekenne ich, daß mir die Wahl zwischen Herrn Dirichlet und Herrn Eisenstein schwer geworden sein würde, da die Arbeiten des letztern in vollem Maße dasselbe Prädicament verdienen, wie die des erstern.*

Jacobi lässt daraufhin seine alte Arbeit aus dem Jahre 1837 über Kreisteilung in Crelles Journal wieder abdrucken und fügt eine Fußnote hinzu, in welcher er Eisenstein des Plagiats bezichtigt: zum Einen soll er sich der Kollegienhefte zu Jacobis Vorlesung über Zahlentheorie bedient haben, zum andern stimme einer von Eisensteins Beweisen des quadratischen Reziprozitätsgesetzes mit seinem eigenen überein, den Legendre in seine Zahlentheorie aufgenommen habe. Auch Cauchy hat diesen Beweis (mehr oder weniger zeitgleich mit Jacobi) veröffentlicht, und keinem der dreien ist aufgefallen, dass es sich dabei im Wesentlichen um den sechsten Gaußschen Beweis handelt; erst Gauß hat Eisenstein dies in seinem Brief vom 23. Juni 1844 an Encke (siehe [24]) wissen lassen:

An der Art wie er sich über die bisherigen Beweise äußert, möchte ich fast vermuthen daß ihm das, was im XVII Band der Pariser Memoires S. 454 steht nicht bekannt geworden ist. Mir selbst ist dieser Band erst in diesen Tagen zu Gesicht gekommen, und die 3te Edition von Legendre „Théorie des Nombres“, auf welche seine Stelle Bezug nimmt, habe ich überall noch nicht gesehen. Machen Sie doch Herrn Eisenstein auf diese Stelle aufmerksam, über welche ich übrigens seinem Urtheile nicht vorzugreifen brauche.

Offenbar hat es Jacobi nicht bei dieser Aktion belassen: In einem Brief an Olfers[3] schreibt Humboldt:

Da ich bestimmt weiß, daß Jacobi und selbst Encke auf die liebloseste Weise dem jungen blutarmen Mathematiker Eisenstein beim Minister Eichhorn zu schaden gesucht, so habe ich mir die süße Freude gemacht, beiden einen neuen Brief von Gauß zu schicken, in dem verbotenus[4] steht (14. April 1846):

Ob ich gleich voraussetzen darf, daß meine Empfehlung dem in Berlin lebenden jungen Mathematiker Doctor Eisenstein, eines Mannes, den ich sehr hochschätze, bei Ihrer Regierung ganz überflüssig ist, so will ich doch nicht unterlassen, es auszusprechen, daß ich seine Begabung wie eine solche betrachte, welche die Natur in jedem Jahrhundert nur wenigen erteilt.

[3] [18, S. 105–106].

[4] Lateinisch für „wortwörtlich“.

Humboldt fährt fort, Jacobi habe

> *seitdem ausgesprochen, daß Gauß ganz herunter und verstandesschwach geworden sei, daß Herr Eisenstein ja nicht daran denken müsse, sich hier als Privatdozent zu habilitieren, sondern nach Halle, oder nach Bonn gehen müsse!!!*

Kommentar zum Beweis des Reziprozitätsgesetzes

Der oben vorgestellte Beweis Eisensteins für das quadratische Reziprozitätsgesetz unterscheidet sich nur formal von seinem bekannten Beweis mithilfe der Sinusfunktion (siehe [3]). Der wesentliche Unterschied der beiden Beweise liegt darin begründet, dass $\sin(qx)$ für ungerade Werte von q ein Polynom in $\sin x$ ist, während $\tan(qx)$ eine rationale Funktion von $\tan x$ ist – der Grund dafür ist, dass die Sinusfunktion eine ganze Funktion ist, während die Tangensfunktion Pole besitzt. Der Eisensteinsche Beweis des biquadratischen Reziprozitätsgesetzes mithilfe elliptischer Funktionen ist daher näher am Beweis mit der Tangensfunktion, denn elliptische Funktionen haben zwangsläufig Pole.

Eisenstein spielt am Ende des § 1 in [3] auf diesen Beweis an; er schreibt nach der Bestimmung einer Konstanten C in seinen Kongruenzen:

> *Will man die Konstante C vermeiden, muss man sich des Tangens anstatt des Sinus bedienen.*

Koschmieder hat einen solchen Beweis mithilfe der Tangensfunktion in [10] gegeben und darüber am 20. September 1961 in Oberwolfach vorgetragen; im Tagungsbericht steht dazu:

> *Eisenstein hat im Crelle Journal 29 (1845) einen Beweis des quadratischen Reziprozitätsgesetzes gegeben, bei dem er eine transzendente Hilfsfunktion benutzt, nämlich die Multiplikation des Sinus herangezogen hat. Er bemerkt dazu, daß der Beweis noch glatter liefe, wenn man statt des Sinus den Tangens gebrauchte. Bisher scheint das nicht geschehen zu sein. Der Vortragende führt diesen Beweis.*

Kommentare zum Beweis des kubischen Reziprozitätsgesetzes

Den Beweis des kubischen Reziprozitätsgesetzes, den Eisenstein am 26. Januar 1845 gefunden hat, hat er ebenfalls nicht veröffentlicht; in seinem Artikel [3, § 3] vom 13. Februar 1845 schreibt Eisenstein, dass der Beweis dem des biquadratischen Reziprozitätsgesetzes ganz analog sei. Eisensteins Beweisidee wurde später von zahlreichen Autoren wieder aufgegriffen und ausgearbeitet, etwa von Hübler

[7], Dantscher[5] [2], Gegenbauer [6], Sbrana [20], Lewandowski [13], Koschmieder [9], Petr [19], Mel'nikov [15] und Kubota [12].

Koschmieder erwähnt in [11], dass Lewandowski in [14] auch das Reziprozitätsgesetz der 9ten Potenzreste mit elliptischen Funktionen behandelt hat; ich habe aber weder diese Arbeit, noch die darauf Bezug nehmende Arbeit von Georg Kantz [8] aus dem Jahre 1929 auffinden können.

Literatur

[1] A. Aigner, *Zahlentheorie*, de Gruyter 1975.

[2] V. Dantscher, *Bemerkungen zum analytischen Beweise des cubischen Reciprocitätsgesetzes*, Math. Ann. **12** (1877), 241–253.

[3] G. Eisenstein, *Applications de l'Algèbre à l'Arithmétique transcendante*, J. Reine Angew. Math. **29** (1845), 177–184; Math. Werke I, 291–298.

[4] G. Eisenstein, *Beiträge zur Theorie der elliptischen Functionen. I. Ableitung des biquadratischen Fundamentaltheorems aus der Theorie der Lemniscatenfunctionen, nebst Bemerkungen zu den Multiplikations- und Transformationsformeln*, J. Reine Angew. Math. **30** (1846), 185–210; Math. Werke I, 299–324.

[5] *Carl Friedrich Gauß Letters*, Constantin Eisenstein [sen.] $\to$ Carl Friedrich Gauß, Berlin, 1852 Nov. 12; https://gauss.adw-goe.de/handle/gauss/2588?.

[6] L. Gegenbauer, *Ueber das cubische Reciprocitätsgesetz*, Wiener Ber. **81** (1880), 1–5.

[7] H. P. Hübler, *Über die kubischen Reste*, Diss. Jena, 1871.

[8] G. Kantz, nach [11] in den Comment. Math. Helvetici **2** (1929), 116–127.

[9] L. Koschmieder, *Über eine besondere äquianharmonische elliptische Funktion und ihre Verwendung in der Zahlentheorie*, Sitzungsber. Berlin. Math. Ges. **21** (1922), 12–15.

[10] L. Koschmieder, *Zu Eisensteins transzendentem Beweis des quadratischen Reziprozitätsgesetzes*, Comment. Math. Helv. **37** (1962/63), 235–239.

[11] L. Koschmieder, *Ein Gefüge von Differentialgleichungen, durch elliptische Funktionen integriert*, Monatsh. Math. **54** (1950), 265–283.

[12] T. Kubota, *Anwendung Jacobischer Thetafunktionen auf die Potenzreste*, Nagoya Math. J.**19** (1961), 1–13.

[13] J. F. Lewandowski, *Über die äquianharmonische Funktion*, Monatsh. Math. Phys. **21** (1910), 155–170.

[14] J. F. Lewandowski, nach [11] in den Jahresberichten des Kaiser-Franz-Josef Landesreal- und Obergymnasiums in Baden bei Wien, **47** (1919), 12 S.

[5] Die Namen Dantscher, Gegenbauer und Lewandowski zeigen das große Interesse, das den niederen Reziprozitätsgesetzen in Österreich entgegengebracht wurde. Auch Franz Mertens [16] hat sich mit analytischen Beweisen des kubischen und biquadratischen Reziprozitätsgesetzes beschäftigt, und Phänomene der Reziprozität spielen in der *Zahlentheorie* [1] Alexander Aigners eine große Rolle. Und dann ist da noch Emil Artin …

[15] G. I. Mel'nikov, *Two proofs of the cubic law of reciprocity with the aid of the theory of elliptic functions* (Russ.), Uchenje Zap. Leningrad (Fiz. Matem. Fak.), **14** (1955).

[16] F. Mertens, *Über die Darstellung der Legendreschen Symbole der biquadratischen, kubischen und bikubischen Reste durch Thetareihen*, Sitz.ber. Akad. Wiss. Wien. **115** (1906), 1339–1360.

[17] E. Neuenschwander, *Lettres de Bernhard Riemann à sa famille*, Cah. Sémin. Hist. Math. **2** (1981), 85–131.

[18] E. W. M. v. Olfers (Hrsg.), *Briefe Alexander v. Humboldts an Ignaz v. Olfers, Generaldirektor der Kgl. Museen*, Nürnberg 1913.

[19] K. Petr, *Über die Eisensteinschen Beweise des Reziprozitätsgesetzes bei den biquadratischen und bikubischen Resten* (Czech.), C. R. Congrès Math. Pays slaves 1929 (1930), 119–128.

[20] U. Sbrana, *Alcune proprietà dell'equazione per la divisione dei periodi di una funziona equianarmonica*, Battaglini Giorn. **42** (1904), 297–311.

[21] P. Ullrich, *Gotthold Eisensteins Exemplar der Gauß'schen Disquisitiones Arithmeticae, erneut betrachtet*, In *Neue Welten. Wilhelm Olbers und die Naturwissenschaften um 1800* (G. Biegel at al., eds.) 2001, 202–221.

[22] P. Ullrich, *Über das Exemplar der "Disquisitiones Arithmeticae" aus dem Besitz von Gotthold Eisenstein*, Mitt. Math. Ges. Hamb. **21** (2002), 35–58.

[23] A. Weil, *On Eisenstein's copy of the Disquisitiones*, Algebraic number theory – in honor of K. Iwasawa, Adv. Stud. Pure Math. **17** (1989), 463–469.

[24] A. Wittmann, *Obgleich und indeßen: Der Briefwechsel zwischen Carl Friedrich Gauß und Johann Franz Encke*, Kessel Verlag 2018.

Adresse des Autors:
Mörikeweg 1, D-73489 Jagstzell, Deutschland
email hb3@uni-heidelberg.de

Internat. Math. Nachrichten
Nr. 247 (2021), 29–30

Franz Kappel 1940–2020

Karl Kunisch
Universität Graz

Franz Kappel, Professor emeritus at the Institute of Mathematics and Scientific Computing at the Karl-Franzens University of Graz, Austria, passed away on April 3rd, 2020, during the early stages of the first local corona wave. He was a pioneer in the field of delay differential equations during the first part of his career and made a substantial move to research at the interface of mathematics and biomedical modelling in the second. He guided an impressive list of young researchers and engaged in university services in all possible ranks.

Professor Kappel was born in Graz, Styria, on February 18th, 1940. In 1963, he obtained his Ph.D. at the University of Graz and in 1970 his habilitation from the Technical University of Graz, where he worked as an assistant professor until 1971. He then moved to Würzburg to work as a professor in the group of Professor H.W. Knobloch. Together they published the monograph "Gewöhnliche Differentialgleichungen" (Ordinary differential equations), which has been a standard text for many generations of students.

I got to know Franz in an off-campus office at the University of Graz in the summer of 1975. He unpacked his books after returning from Würzburg and I was looking for a Ph.D. advisor. I then had the tremendous fortune to be one of his pupils who he instructed with judicious care mathematically and who he thoughtfully guided towards scientific life personally.

At the beginning of Franz's academic career, stability theory of dynamic systems in the spirit of Liapunov's direct method, and the Barbashin-Krasovskij-LaSalle invariance principle were in the center of his attention. He then moved on to the investigation of functional (delay-) differential equations first systematically exploiting Laplace-transform techniques and establishing an elementary divisor theory, then turning to approximation theory. Let me mention the seminal paper with H.T. Banks (with whom Franz was a good friend of the same age, who passed away very recently as well) on spline approximation of delay differential equations, the work on Riccati equations with D. Salomon, and the abstract Trotter-Kato based schemes jointly with K. Ito.

All along Franz had a strong interest in mathematical modelling. He was the first to teach classes at our department concentrating on bio applications. Much earlier

ISSN 0020-7926

than a large part of the community he investigated population models including features such as size or age structure. In the late 1990ies, he distinctly moved towards cardiovascular modelling including the respiratory system. His research on modelling and subsequently controlling the dialysis process led to a most fruitful collaboration with the Renal Research Institute in New York, which he frequently visited throughout his retirement. Franz enjoyed these visits since they provided an opportunity to push the limits of detailed modelling as demanded by personalized medicine. They also provided an opportunity to revisit the East Coast which he appreciated since his sabbatical at Brown University in 1977.

It came naturally with Franz Kappel that he provided significant services to the scientific community. Many of his colleagues in control or semi-group theory will remember the workshops organized in Retzhof, Vorau, and Maria Trost, who brought together leaders in their fields on a world-wide scale. Franz also engaged strongly in ASEA Uninet, advising Ph.D. students from Indonesia, Malaysia, and the Philippines, taught courses in their countries, and worked towards bridging cultural gaps. In Styria throughout many years, he offered a "modeling weak" (Modellierungswoche) for upper level highschool students, in an effort to raise awareness for the importance of mathematics in every day life. In all these activities, as well as in many others, he had the strong support from his wife Grit. Finally, he was also department head for 18 years, Dean of the Faculty of Science for 6, and Vicerector for Finance for four years.

Franz Kappel was an inspiring person on diverse levels who is greatly missed by the Institute of Mathematics at the University of Graz, which he shaped for many years, and by his friends and colleagues worldwide.

A similar version of this article first appeared on SIAM News Online.

Author's address:
Institut für Mathematik und Wissenschaftliches Rechnen, Universität Graz, Universitätsplatz 3, A-8010 Graz, Austria
email karl.kunisch@uni-graz.at

Internat. Math. Nachrichten
Nr. 247 (2021), 31–33

Nonlinear Water Waves: A Challenge for Mathematics

Joachim Escher
Leibniz University Hannover

The Austrian Science Fund (FWF) has decided to award the Wittgenstein Prize 2020, Austria's most important research award, to Prof. Dr. Adrian Constantin, University of Vienna, for his groundbreaking and seminal contributions to mathematical hydrodynamics and its applications to water wave phenomena.

In 1755 Leonard Euler was the first to write down a complete system of partial differential equations describing exactly the motion of an inviscid incompressible fluid, cf. [15]. During the 18th century and to the present day Euler's equations of hydrodynamics have been a wellspring, even a cornucopia for many branches in mathematics, such as partial differential equations, complex analysis, dynamical systems, functional analysis, and geometrical analysis among others.

Since Euler there are few mathematicians who contributed in a fundamental way to so many areas of mathematical hydrodynamics as Adrian Constantin has. His first profound results were related to the analysis and the derivation of shallow water wave equations which go beyond the famous Korteweg-de Vries equations dating back to 1895. A major challenge in this endeavour was to find a Hamiltonian system describing one-dimensional shallow water waves, which is completely integrable and admits solitons as well as wave breaking solutions. After decades of mathematical research on the Korteweg-de Vries equation, this task was clearly formulated by G.B. Whitham, FRS: "Although both breaking and peaking, as well as criteria for the occurrence of each, are without doubt contained in the equations of the exact potential theory (i.e. the Euler equations, J.E.), it is intriguing to know what kind of simpler mathematical equation could include all these phenomena", cf. [17].

Constantin addressed this challenge and proved in a serious of seminal papers that all these properties are realized in the so-called Camassa-Holm equation, see [2, 3, 4, 7, 12]. Subsequently he further contributed in a fundamental way to the understanding of the geometric re-expression of these equations as geodesic flows

ISSN 0020-7926

on suitable Fréchet-Lie groups of diffeomorphisms of the circle, cf. [11, 13].

Frequently Constantin is ahead of his time. He is interested in fundamental questions nature poses to us and he has the capacity and the creative power to address the most difficult and deep problems. Once he obtains fundamental insight into a problem he often moves on and let others till the ground he has uncovered.

Mathematical hydrodynamics and complex analysis have been closely connected since the genesis of the latter and a multitude of beautiful and important results for irrotational flows have their source in this liaison. However flows whose vorticity does not vanish exactly, provide an altogether more difficult challenge in that most of the powerful tools from complex analysis are no longer available – even when the vorticity is constant, not to mention discontinuous vorticity distributions, which are particular significant in real world applications. In addition, Helmholtz vorticity theorem makes clear that irrotational flows describe an exceptional situation. Despite these challenges Constantin has made many fundamental contributions to the analysis of irrotational and rotational flows. Pre-eminent among these are the proof of the existence of exact large-amplitude steady periodic water waves with general vorticity distributions together with Walter Strauss [14], the description of the particle trajectories in Stokes waves [5], and the proof of the analyticity of the streamlines of periodic traveling free surface water waves with vorticity [8], generalizing a famous result for irrotational flows due to Hans Lewy [16].

In recent years Constantin started to address research questions that arise in the study of geophysical flows, specifically waves and currents in the ocean and in the atmosphere. Jointly with Robin S. Johnson he studied such flows in rotating spherical coordinates, highlighting several pitfalls of the usual approach that consist in using approximations formulated in flat-space geometry, cf. [11, 13].

Recalling the words of Arnold & Keshin in [1]: "Hydrodynamics is one of those fundamental areas where progress at any moment can be regarded as a standard to measure the real success of mathematical science. ... In spite of all this acknowledged success, hydrodynamics with its spectacular empirical laws remains a challenge for mathematicians", in this sense, Constantin's work on nonlinear wave phenomena represents a further significant step to increase and deepen our understanding of challenging problems in hydrodynamics.

Adrian Constantin's successful and extremely productive work has received many awards. In 2010 he was awarded an Advanced Grant of the European Research Council. The previous year he received the Fluid Dynamics Research Prize of the Japan Society for Fluid Mechanics. The Alexander-von-Humboldt-Stiftung honored him with the Friedrich Wilhelm Bessel Research Award in 2007 and in 2005 the Royal Swedish Academy of Sciences awarded him the Göran Gustafsson Prize. In the year 2012 Constantin gave a plenary talk at the European Congress of Mathematics in Krakau. The Wittgenstein Prize 2020 of the Austrian Science Found honors an outstanding scientist of highest creativity and with a boundless passion for mathematics and its applications.

References

[1] V. I. Arnold & B. A. Keshin: *Topological Methods in Hydrodynamics*, Springer, New York, 1998.

[2] A. Bressan & A. Constantin: Global conservative solutions of the Camassa-Holm equation, Arch. Ration. Mech. Anal. 183 (2007), 215-239.

[3] A. Constantin, On the inverse spectral problem for the Camassa-Holm equation, J. Funct. Anal. 155 (1998), 352-363.

[4] A. Constantin: Existence of permanent and breaking waves for a shallow water equation: a geometric approach, Ann. Inst. Fourier (Grenoble) 50 (2000), 321-362.

[5] A. Constantin: The trajectories of particles in Stokes waves, Invent. Math. 166 (2006), 523-535.

[6] A. Constantin: Frictional effects in wind-driven ocean currents, Geophy. Astrophys. Fluid Dynamics, 115 (2021), 1-14.

[7] A. Constantin & J. Escher: Wave breaking for nonlinear nonlocal shallow water equations, Acta Math. 181 (1998), 229-243.

[8] A. Constantin & J. Escher: Analyticity of periodic traveling free surface water waves with vorticity, Ann. of Math. 173 (2011), 559-568.

[9] A. Constantin & R. S. Johnson: On the nonlinear, three-dimensional structure of equatorial oceanic flows, J. Phys. Oceanography 49 (2019), 2029-2042.

[10] A. Constantin & R. S. Johnson: On the propagation of waves in the atmosphere, Proc. Royal Soc. A 477 (2021), 20200424.

[11] A. Constantin & B. Kolev: Geodesic flow on the diffeomorphism group of the circle, Comment. Math. Helv. 78 (2003), 787-804.

[12] A. Constantin & D. Lannes: The hydrodynamical relevance of the Camassa-Holm and Degasperis-Procesi equations, Arch. Ration. Mech. Anal. 192 (2009), 165-186.

[13] A. Constantin & H. P. McKean, A shallow water equation on the circle, Comm. Pure Appl. Math. 52 (1999), 949-982.

[14] A. Constantin & W. Strauss: Exact steady periodic water waves with vorticity. Comm. Pure Appl. Math. 57 (2004), 481-527.

[15] L. Euler: Principes généraux du mouvement des fluides, Mémoires de L'Académie Royale des Sciences et des Belles-Lettres de Berlin 11 (4 September 1755, printed 1757), 217-273; reprinted in Opera Omnia ser. 2 12, 219-250.

[16] H. Lewy: A note on harmonic functions and a hydrodynamical application, Proc. Amer. Math. Soc. 3 (1952), 111-113.

[17] G. B. Whitham: *Linear and Nonlinear Waves*, Wiley, New York, 1974.

Author's address:
Institut für Angewandte Mathematik, Leibniz Universität Hannover, Welfengarten 1, D-30167 Hannover, Germany
email escher@ifam.uni-hannover.de

Internat. Math. Nachrichten
Nr. 247 (2021), 35–37

Nicht was real ist, sondern was bleibt, sollte gefragt werden

Rudolf Taschner
TU Wien

Adam Beckers Buch „Was ist real?“ befasst sich mit Persönlichkeiten, die sich um die Deutung der Quantenphysik bemühten, sowie mit dem historischen Umfeld. Der Autor entwirft dabei ein Panorama einer akademischen Debatte, die an die Scholastik mit ihrem Universalienstreit und den Schulen des Nominalismus, des Realismus, des Konzeptualismus sowie aller dazwischenliegenden Facetten erinnert. Den unbestreitbaren Vorzug dieses Buchs erblickt der Rezensent darin, dass sich der Autor bemüht, möglichst viele Stimmen zu Wort kommen zu lassen, den beklagenswerten Nachteil jedoch darin, dass man dieses Buch nicht verorten kann:

Es ist kein historisches Werk. Dazu fehlt dem Autor die schon von Tacitus verlangte Nüchternheit, Sachlichkeit und Unparteilichkeit; Becker schreibt nicht „sine ira et studio“. Dass sich seine abschätzigen Urteile über Vertreter der sogenannten Kopenhagener Interpretation der Quantentheorie zuweilen sogar als untergriffig ausnehmen, schmerzt beim Lesen.

Es ist kein philosophisches Werk. Dazu fehlt die substanzielle Auseinandersetzung mit den Konzepten des Realismus und des Idealismus, an denen sich die vom Autor vorgestellten Deutungsversuche zu messen hätten. Die im Buchtitel vorgelegte Frage „Was ist real?“ nimmt der Autor, allen wortreichen Beteuerungen zum Trotz, leider nicht ernst genug.

Es ist kein für interessierte Laien verfasstes Werk, obwohl dem Autor dies vielleicht vorschwebte. Dazu hätte er sich um die Klärung der von ihm verwendeten Fachbegriffe bemühen müssen und nicht diese wie die Kulissen eines potemkinschen Dorfs ohne Unterlass hin- und herschieben sollen. Damit erzeugt der Autor bestenfalls Halbbildung, die bekanntlich schädlicher als Unbildung ist.

Es ist kein physikalisches Werk. Dazu fehlt im gesamten Buch jener unverzichtbare Eckstein der Physik, ohne den sie nicht entfaltet werden kann: die mathematische Sprache. In gewisser Hinsicht wird dieser Lapsus vom Autor sogar als Vorteil

ISSN 0020-7926

hingestellt, wenn er zum Beispiel behauptet: „Die Mathematik hinter der Quantenphysik ist unvertraut und abstrus.“ Auch spricht er bei der Allgemeinen Relativitätstheorie von einer „seltsamen Mathematik“ – das Eigenschaftswort seltsam hätte Carl Friedrich Gauß und Bernhard Riemann, den Begründern der Differentialgeometrie, wohl kaum gefallen.

Eine der wohl profundesten Aufbereitungen der Quantenphysik auf mathematischer Basis findet man im dritten Band des epochalen Lehrbuchs der mathematischen Physik von Walter Thirring. Die Frage, was „real“ sei, stellt er als mathematischer Physiker naturgemäß nicht. Aage Petersen zitiert in eben diesem Sinne Niels Bohr: „Es gibt keine Quantenwelt. Es gibt nur eine abstrakte quantenphysikalische Beschreibung. Es ist falsch, zu glauben, dass die Aufgabe der Physik darin besteht, herauszufinden, wie die Natur ist. Der Physik geht es darum, was wir über die Natur sagen können.“

Natürlich sind mit der abstrakten Mathematik Modellvorstellungen einer „Quantenwelt“ verwoben. Besonders anschaulich schildert eine solche der eben genannte, höchst abstrakt denkende Thirring, als bei einem Gespräch mit ihm und Werner Heisenberg die Sprache auf das Photon, das Lichtquantum kam: „Heisenberg philosophierte vor sich hin: ‚Ein Photon, wie kann ich mir das vorstellen? Es ist ein Teilchen, das immer mit Lichtgeschwindigkeit fliegt, wird also durch die Lorenzkontraktion platt wie eine Wanze. Und wie groß ist jetzt der Durchmesser dieses Scheibchens? Da es mit Elektronen wechselwirkt, wird sein Radius etwa eine Compton-Wellenlänge des Elektrons sein, also zehn hoch minus elf Zentimeter. (Heisenberg dachte in Zentimeter, nicht in Meter.) Allerdings ist der Wirkungsquerschnitt für Streuung an Elektronen nicht zehn hoch minus 22, sondern zehn hoch minus 26 Quadratzentimeter, also muss es sehr durchsichtig sein. Denken wir uns somit, das Photon wäre ein ganz dünnes, durchsichtiges Plättchen, zehn hoch minus elf Zentimeter breit.‘ Natürlich verstieß Heisenberg dabei gegen ein Gebot der Quantenteilchen: ‚Du sollst dir von mir kein Bildnis machen‘, aber ich wurde dabei von dieser Unart, in Bildern zu denken, infiziert, und sie hat mir oft genützt.“

Die „nützliche Unart“, in Bildern zu denken, sich Modelle zu machen, die selbstverständlich von der anschaulichen Alltagswelt herrühren, sind ein viel zu schwaches Fundament für den faustischen Versuch, zu erkennen, was die Welt im Innersten zusammenhält. Doch mit nichts anderem als solchen Modellvorstellungen argumentiert Becker. Er verwechselt Modell mit Realität und will nicht wahrhaben, dass Bilder nichts anderes sind als Sprossen einer zur Erkenntnis führenden Leiter, von der Ludwig Wittgenstein vorschlägt, es müsse sie der nach Wahrheit Suchende „wegwerfen, nachdem er auf ihr hinaufgestiegen ist“. Statt sich der von Scheinproblemen durchwachsenen Frage „Was ist real?“ zu stellen, erhebt sich die grundsätzlichere Frage „Was bleibt?“

Sicher „bleiben“ nicht die zwar die Intuition beflügelnden, aber dennoch mit unvermeidlichen Mängeln behafteten Modellvorstellungen, sei es die von Bohr und

Heisenberg stammende Kopenhagener Deutung, seien es die von Louis de Broglie und David Bohm vorgeschlagenen Führungswellen, sei es die von Hugh Everett erdachte Viele-Welten-Interpretation. Und es ist müßig, wie Becker darüber zu streiten, welche der Modellvorstellungen mit der Wirklichkeit übereinstimmen. Denn die banale Einsicht lautet, dass dies bei keiner der Fall ist, getreu dem bekannten Wort des Statistikers George Box: „All our models are wrong, but some are useful."

Sicher „bleibt" die Mathematik der Quantenphysik. Sie ist, so gesehen, das einzig „Reale". Ein erstaunlicher Befund, da Mathematik eine glatte Erfindung des menschlichen Geistes ist. Es ist dies jenes Erstaunen, das sich in Eugene Wigners berühmten Wort von der „unreasonable effectiveness of mathematics in the natural sciences" gespiegelt findet.

Literatur

[1] Adam Becker: Was ist real? Das ungelöste Problem der Quantenphysik. Taschenbuch, 368 Seiten, Springer-Verlag 2021

Adresse des Autors:
Institut 101 der TU Wien, Wiedner Hauptstraße 8-10, A-1040 Wien, Austria
https://www.rudolftaschner.at/, email taschner@math.space.or.at

Buchbesprechungen

J. Beineke, J. Rosenhouse (eds.): The Mathematics of Various Entertaining Subjects (R. GERETSCHLÄGER) 39

A. Crannell, M. Frantz, F. Futamura: Perspective and Projective Geometry (R. GERETSCHLÄGER) 40

J. Beineke, J. Rosenhouse (eds.): The Mathematics of Various Entertaining Subjects. Volume 3. The Magic of Mathematics. Princeton University Press, 2019, 352 S. ISBN 978-0-69118-258-2 P/b $ 49,95.

In diesem dritten Band der Reihe präsentieren die Herausgeber eine vielfältige Mischung an Themen, die, von der Welt der Unterhaltungsmathematik ausgehend, Anregungen zu hochinteressanten Forschungsergebnissen anbieten. Die Kapitel dieses Buchs sind das Ergebnis von Beiträgen zur MOVES-Konferenz 2017 (Mathematics of Various Entertaining Subjects) und setzen so das Format der ersten beiden Bände aus den Jahren 2015 bzw. 2017 fort. Sie behandeln so unterschiedliche Themen wie Zauberkunststücke, die sich aus zahlentheoretischen Erkenntnissen ergeben, Wegminimierungsaufgaben, die sich aus der Betrachtung eines Brettspiels ableiten, oder algorithmische Komplexitätsaspekte des Damespiels.

Die Lektüre dieses Buchs kann all jenen sehr empfohlen werden, die sich für anspruchsvolle Themen aus der Unterhaltungsmathematik begeistern. Es wäre aber auch jenen Skeptikern sehr ans Herz zu legen, die die Meinung vertreten, dass es sich bei der Unterhaltungsmathematik nicht um „echte“ Mathematik handelt. Die Themen, die in diesen Arbeiten behandelt werden, weisen zum Teil in sehr tiefe Bereiche des mathematiche Denkens und zeigen auf hervorragende Weise einige Schnittstellen von mathematischen „Spielereien“ zur wissenschaftlichen Forschung.

R. Geretschläger (Graz)

A. Crannell, M. Frantz, F. Futamura: Perspective and Projective Geometry. Princeton University Press, 2019, 296 S. ISBN 978-0-69119-656-5 P/b $ 49,95.

In diesem neuen Lehrbuch zur dreidimensionalen Geometrie, das in erster Linie für Studierende aus dem Bereich der bildenden Kunst gestaltet wurde, ist der Autorengruppe ein durchaus bemerkenswerter Spagat gelungen: Einerseits findet sich hier ausreichend Raum für einen intuitiven Zugang zur Perspektive, aber dem gegenüber steht der mathematisch saubere und vollständige Aufbau des behandelten Stoffs. Das hat wohl zur Folge, dass man sich ausreichend Zeit für die Arbeit mit diesem Buch lassen sollte, aber der Zeitaufwand verspricht auch entsprechend belohnt zu werden.

Die 13 Arbeitskapitel führen, von einem naiven Zugang zur Perspektive ausgehend, über die Koordinatisierung der Ebene und des Raumes und der Bedeutung des Doppelverhältnisses zu einer Auseinandersetzung mit synthetischer und analytischer Beweistechnik, etwa anhand des Satzes von Desargues oder dem Arbeiten mit homogenen Koordinaten. Dazu kommen einige abstrakte Betrachtungen zu topologischen Ideen sowie praktische Dinge, wie eine kurze Abhandlung über das korrekte Schreiben eines mathematischen Beweises oder der praktischen Arbeit mit Geogebra.

Alles in allem ergibt das eine sehr ansprechende Fusion von mathematischer Vollständigkeit mit anschaulichem Zugang, die anspruchsvollen Kunststudierenden einen exzellenten Zugang zum Thema ermöglichen sollte.

R. Geretschläger (Graz)

Women in Mathematics

Interview with Michaela Szölgyenyi

Michaela Szölgyenyi is a full-professor for Stochastic Processes at the Department of Statistics, University of Klagenfurt since 2018; in 2020 she became head of department. Michaela is mainly working on analysis and numerical methods for stochastic differential equations and stochastic optimal control. In 2020 the University of Klagenfurt was granted the FWF doc.funds doctoral school Modeling – Analysis – Optimization of discrete, continuous, and stochastic systems of which she is the coordinator. Michaela did her PhD in Mathematics at JKU Linz in 2015 and Post-docs at WU Vienna and at ETH Zurich.

(c) Christina Supanz

How did you discover your attraction to Maths?

I think I just always liked it. Already as a four-year-old my grandfather played school with me, teaching me basic calculus.

Did you or do you have any role models?

During my career I met several great female mathematicians. At the moment I would call Barbara Kaltenbacher my role model. She is incredibly successful in research, extremely efficient, and still finds time for being very active in our departments' life. I deeply admire her.

Did you have support (encouragement) from some mentor during your career?

I think the most important time in my career development was my time at WU Vienna. My mentor there was Rüdiger Frey, who was always very clear and honest with me in order to push me forward. For example, if you like the place where you are and your dear mentors tell you that you should apply to another great place abroad, then this might feel hard in the first place, but they are probably right. Besides a very good personal relationship we had, Rüdiger did a great deal in fostering me.

Why did you choose to work on stochastic differential equations and related topics?

I did my PhD on stochastic optimal control problems in insurance mathematics. While solving those problems, issues with existence and uniqueness of solutions to the underlying SDEs emerged. So I dived into those questions and soon became hooked also with numerics of SDEs and simulation methods.

What scientific achievements are you most proud of?

I am happy that nowadays several international research groups work on topics close to mine and also make use of methods from my papers. In addition, I am proud that we have been granted the first FWF doc.funds doctoral program at our university and that so many great young people came to Klagenfurt to work with us on this project.

You received a professorship only three years after the PhD graduation. What is the secret of your success?

I worked hard on my research papers, gave numerous talks at international conferences and in my departments' seminars, asked for feedback from my peers and mentors, and applied for third party funding already at an early stage of my career. In addition, I was involved in a lot of teaching and in university management as a member of WU's academic senate, where I learned a lot. I left JKU Linz shortly after my PhD defense, went on to WU Vienna and then after two and a half years to ETH Zurich, where I enjoyed a great research environment. So there are no secrets involved, just commitment, mobility, hard work, and great mentors.

What are the challenges of becoming a professor at such an early age stage of the career and how do you deal with them?

Independent of the age is the challenge that you suddenly have much more duties in teaching and university administration that reduces the time you can spend on your beloved research.

You are the leader of a prestigious FWF doc.funds doctoral school at the University of Klagenfurt. Could you tell us more about this achievement?

After being in Klagenfurt for only half a year, the faculty of the Departments of Statistics and Mathematics at University of Klagenfurt decided to apply for an FWF doc.funds project. As my field of research is well-connected to the other fields, I was asked if I would take over the role of the coordinator (project lead), to which I happily agreed. In February 2020, only days before the first lockdown, four female professors from Klagenfurt took part in the FWF hearing – my dear colleagues Barbara Kaltenbacher, Angelika Wiegele, our vice-rector for research Friederike Wall, and me. It was an awesome moment when the FWF president called me in March to tell me that we were successful! In the project we work on the proposed problems in a multi-perspective way, that is by joining different mathematical sub-disciplines. We do this together with 14 internationally hired PhD students, currently over 80 per cent female. It is extremely positive that so many fantastic young researchers fill up our departments with their spirits!

There are many ups and downs for young researchers, especially in Mathematics and especially for women. Do you have any special advice for students or postdocs in this regard?

It is hard to move all the time and once one is settled, move again. This also takes time – time that one would love to spend for research. Also it is hard to settle and it is hard for the relationships you have. But these issues are gender independent. Women suffer – more often than average – from the imposter syndrome. If you don't know it, Google it! And then stop it!

Interview organized by Sylvia Frühwirth-Schnatter (Vienna University of Economics and Business) and Elena Resmerita (University of Klagenfurt).

Report on the First Austrian Day of Women in Mathematics

Over the last months, on the initiative of Elena Resmerita (University of Klagenfurt), a core team of eight female mathematicians from universities in Graz, Innsbruck, Klagenfurt, Linz, Vienna, and IST Austria has founded the Austrian Association of Women in Mathematics (A2WiM).

Our newly created network aims to increase the visibility of the achievements of female mathematicians working at Austrian institutions and to foster interdisciplinary collaborations as well as mutual support such as career advice and mentoring. We also hope to inspire more girls to discover and pursue their interest in mathematics, since the gender gap in this area is still significant.

The "First Austrian Day of Women in Mathematics" https://sites.google.com/view/adwim/ was the first scientific event of the A2WiM network. It took place as a virtual meeting on February 23, 2021. The intention was to include a broad variety of topics and to highlight the achievements of female mathematicians at different career stages working at Austrian universities. Considering the attendance of 150 international participants from all genders and generations, we believe that this first event was a success. The six scientific talks, held by two advanced speakers, two post-doctoral students, and two PhD students, led to vivid discussions and interdisciplinary exchanges as a testimonial of interest and enthusiasm. The scientific program was framed by an informative talk on women in mathematics and a panel discussion that addressed concrete questions such as the work-life-family balance in an academic career and the importance of mobility, early support, and role models. The event was completed by an informal get-together.

An A2WiM questionnaire accompanied the “First Austrian Day of Women in Mathematics” to reveal the expectations of the participants from the newly created network. The outcome of this poll showed a major interest in regular scientific meetings and networking events such as seminars on career paths and opportunities. Other welcomed activities were relaxed after-work meetings, mentoring programmes, and an informative newsletter. Respondents also supported the active engagement of A2WiM in improving the national as well as international visibility of women in mathematics and in portraying the history of female mathematicians in Austria.

At present, we are organizing the scientific minisymposium “Connecting young researchers by networks” and the workshop “Get-together and career advice for young mathematicians” at the DMV-ÖMG annual meeting https://www.uni-passau.de/dmv-oemg-2021/. The “Second Austrian Day of Women in Mathematics” is planned for February 2022. Furthermore, we are running virtual Friday afternoon tea time meetings to offer a platform for scientific and social exchange. In the international perspective, we intend to intensify our contacts with befriended networks such as “European Women in Mathematics (EWM)” https://www.europeanwomeninmaths.org/.

Currently, we are setting up a website. If you would like to receive our newsletter or be involved in our activities, please

email *anastasia.molchanova@univie.ac.at*.

Forward-looking, we hope that numerous researchers of all genders will join A2WiM, and we welcome all contributions that support the aims of our network.

Hana Dal Poz Kouřimská (IST Austria)
Eranda Dragoti-Cela (Graz University of Technology)
Anastasia Molchanova (University of Vienna)
Mechthild Thalhammer (University of Innsbruck)

Nachrichten der Österreichischen Mathematischen Gesellschaft

Leo Remmel 1926–2020

Dr. Leo Remmel ist im März 2020 verstorben. Dr. Remmel hat 1977 mit der Doktorarbeit *Modalitätentheorie* am Institut für Logistik bei Univ.-Prof. DDr. Curt Christian promoviert. Er war Mitglied der ÖMG seit 1977.

Neue Mitglieder

Kollosche David, Univ.-Prof. Dr. – Institut für Didaktik der Mathematik, Universität Klagenfurt, 9020 Klagenfurt. geb. 1984. Studium des Lehramts für das Gymnasium an der Universität Potsdam und der University of Edinburgh von 2003-2008, Promotion in Didaktik der Mathematik in Potsdam 2014, Postdoc und Vertretungsprofessor in Potsdam und in Frankfurt von 2014-2017, Hochschulprofessur für Mathematik/Primarstufe an der PH Vorarlberg von 2017-2019, Universitätsprofessor für Didaktik der Mathematik an der Universität Klagenfurt seit 2019. email *david.kollosche@aau.at* https://www.aau.at/didaktik-der-mathematik/team/kollosche

Hollnbuchner Michael – Axbergerstr. 3, 4072 Alkoven. geb. 2003. Teilnehmer an der MEMO 2019 sowie der IMO 2020 und 2021. Gewinner des Bundeswettbewerbs der ÖMO 2021. email *michael.hollnbuchner@gmx.at*

Deng Zirui – Fleischmanng. 7, 1040 Wien. geb. 2004. Schüler am Wiener Gymnasium. 1. Preis beim Bundeswettbewerb der ÖMO 2021. email *zrd534@gmail.com*

Obermaier Doris – Arsenal Obj. 16/43, 1030 Wien. geb. 2003. Teilnehmerin an der IMO sowie der EGMO 2020 und 2021. Gewinner des Bundeswettbewerbs der ÖMO 2021. email *doris.obermaier2003@gmail.com*